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On a certain change of affine connections on an almost quaternion manifold

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§ 1. Introduction

Let (M, V) be an almost quaternion manifold[1] of dimension $4m$ $(\geqq 8)$, i.e., a manifold M which admits a 3-dimensional vector bundle V consisting of tensors of type (1, 1) over M satisfying the following condition: In any coordinate neighborhood U of M, there is a local base $\{F, G, H\}$ of V such that

$$\begin{cases} F^2 = G^2 = H^2 = -I, \\ FG = -GF = H, \quad GH = -HG = F, \quad HF = -FH = G, \end{cases}$$

where I is an identity tensor field of type (1, 1) on M. Such a local base $\{F, G, H\}$ of V is called a canonical local base of V in U (cf. [2][2]). We shall discuss in the local and use this canonical local base of V. For convenience sake, we put $J_1 = I$, $J_2 = F$, $J_3 = G$ and $J_4 = H$.

We now consider an affine connection Γ and a curve $C = x(t)$ on an almost quaternion manifold (M, V) satisfying

$$\nabla_{\dot{x}(t)} \dot{x}(t) = \sum_{i=1}^{4} \phi_i(t) J_i \dot{x}(t)$$

where $\dot{x}(t)$ is the vector tangent to C at the point $x(t)$, $\phi_i(t)$ $(i = 1, \cdots, 4)$ are certain functions of the parameter t and ∇ is an operator of covariant differentiation with respect to Γ. Such a curve will be called a Q-planar curve. The purpose of this paper is to prove the following theorem conjectured in the previous paper ([1, p. 242]):

THEOREM. *In an almost quaternion manifold (M, V) of dimension $4m$ $(\geqq 8)$, affine connections Γ and $\bar{\Gamma}$ have all Q-planar curves in common if and only if there exist local 1-forms ψ_i $(i = 1, \cdots, 4)$ on M satisfying*

$$(1) \qquad S(X, Y) + S(Y, X) = \sum_{i=1}^{4} \left\{ \psi_i(X) J_i Y + \psi_i(Y) J_i X \right\},$$

1) Throughout this paper, we assume that manifolds, tensor fields, curves and affine connections are differentiable and of class C^∞.

2) Numbers in brackets refer to the references at the end of the paper.