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On the existence of p-blocks with given defect groups

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1. Introduction.

In [2] Brauer and Fowler proved the following theorem;

THEOREM 1. (Brauer-Fowler (5E), [2]). *Let G be a finite group of even order. Suppose that there exists a conjugate class K of involutions of G and a p-subgroup P such that $z^{x}\neq z^{-1}$ for every $x\in K$, $z\in P^{\#}$, then there exists an irreducible complex character χ such that $p^{c-d}|\chi(1)$, where $p^{c}=|P|$, $p^{d}\top|C_{G}(x)|$.*

Theorem 1 gives us a sufficient condition for the existence of a p-block of defect 0. We can also find in Theorem 1 a sufficient condition for the existence of a p-block with defect group D. Indeed in 2. we shall generalize Theorem 1, and improve the results of N. Ito (Lemma 1, [5]), P. Fong (Theorem (1 G), [4]) and Y. Tsushima (Theorem 1, [7]). In 3. we shall apply our results to a (p, q)-group which shall be defined bellow.

Notations. G denotes always a finite group in this paper. Let us set $|G|_{p}=|P|$ for some $P\in \mathrm{Syl}_{p}(G)$. By Irr (G) we shall mean the set of all irreducible complex characters of G. We denote $p^{d}\top n$, when $p^{d}|n$ and $p^{d+1}\nmid n$ for a prime number p and integers d, n. Let K be a conjugate class of G. We denote by $D(K)$ a p-defect group of K, i.e. a Sylow p-subgroup of $C_{G}(x)$ for some $x\in K$. Let B be a p-block of G. We also denote by $D(B)$ a defect group of B. For distinct prime numbers p, q we call G a (p, q)-group when G satisfies the following condition: p and q are in $\pi(G)$, and $C_{G}(x)$ is a q'-group for every p-element x of G. By a p-element we shall mean a non-trivial p-element. For a fixed prime number p $\mathfrak{p}$ is a prime ideal divisor of p in the field $Q(\varepsilon)$ where ε is a primitive $|G|$-th root of unity. Other notations are standard.

2. A generalization of Theorem 1.

THEOREM 2. *Let D be a p-subgroup of G. Suppose there exist conjugate classes $K_{1}, \cdots, K_{r}$ of G such that $D(K_{i})=D$, $i=1, \cdots, r$, and xy^{-1} is not a p-element for every $x, y\in K_{1}\cup\cdots\cup K_{r}$, then there exist p-blocks $B_{1}, \cdots, B_{r}$ of G which satisfy the following conditions;*