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On cut loci and first conjugate loci of the irreducible symmetric R-spaces and the irreducible compact hermitian symmetric spaces

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Let M be a compact riemannian symmetric space. If M is simply connected, R. J. Crittenden has shown that the tangent cut locus of a point coincides with the first tangent conjugate locus of that point. (See. e.g. [4], [5], [9]).

In this paper we study relations between the cut locus and the first conjugate locus in the case when M is an irreducible compact hermitian symmetric space or an irreducible symmetric R-space. The irreducible symmetric R-spaces are compact riemannian symmetric spaces such as $O(n)/O(m)\times O(n-m)$, $U(n)$, $SO(n)$, $U(2n)/Sp(n)$, $SO(n)\times SO(m)/S(O(n)\times O(m))$ and $T^1\times E_6/E_4$, which are not necessarily simply connected.

In section 1, we give basic notation concerning symmetric spaces and prepare three propositions which will play important roles in section 2.

In section 2, we determine the first tangent conjugate loci and the tangent cut loci for the irreducible compact hermitian symmetric spaces and the irreducible symmetric R-spaces.

In section 3, we culculate the diameters and the injectivity radius of the irreducible compact hermitian symmetric spaces and the irreducible symmetric R-spaces. We study also the closed geodesics in these spaces.

1. Preliminaries.

1.1. Let (G, K) be a compact riemannian symetric pair, which is defined by the following: a) a compact Lie group G and a closed subgroup K of G, b) an involutive automorphism ι of G such that $G_\iota^0\subset K\subset G_\iota=\{g\in G;\ \iota(g)=g\}$, where G_ι^0 is the identity component of G_ι, and c) a G-invariant riemannian structure $\langle\ ,\ \rangle$ on $M=G/K$.

Let $\mathfrak{g}$ (resp. $\mathfrak{k}$) be the Lie algebra of G (resp. K) and put $\mathfrak{p}=\{X\in\mathfrak{g};\ (d\iota)X=-X\}$. Then we have the decomposition $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$. Take a maximal abelian subspace $\mathfrak{a}$ in $\mathfrak{p}$, and denote by $\mathfrak{r}$ the restricted root system with respect to $\mathfrak{a}$, by $\mathfrak{r}^+$ the set of positive roots in $\mathfrak{r}$ with respect to a linear order in $\mathfrak{a}$. Then we have the following decompositions of $\mathfrak{k}$ and $\mathfrak{p}$.