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A family of hypersurfaces in S^{n+1} defined by the harmonic conjugate relation

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Introduction.

In the present paper, the author will study certain hypersurfaces of the $(n+1)$-dimensional unit sphere S^{n+1} which are defined by the harmonic conjugate relation with respect to it as a quadratic hypersurface in R^{n+2}. Here, the harmonic conjugate relation is used in the sense as follows: a point x in R^{n+2} is called *harmonic conjugate* to a point y in R^{n+2} with respect to S^{n+1}, if x is on the polar hyperplane of y with respect to S^{n+1}.

The motivation of the introduction of such hypersurfaces of S^{n+1} is due to his work [3] in which he has investigated minimal hypersurfaces of S^{n+1} with three principal curvature fields and tried to find out examples of such hypersurfaces. He succeeded in constructing such hypersurfaces of special type (Theorem 4 in [3]) under certain conditions for three tangent vector fields of them determined corresponding to these principal curvature fields. In order to find out examples of such minimal hypersurfaces of S^{n+1} without the above mentioned conditions, the author payed his attention to the family of hypersurfaces dealed with in the present paper which were perceived through the properties of the examples in [3] and will try to find out minimal hypersurfaces of this kind out of these families. And, he will show naturality of the examples in [3] by Theorem 6 in some sense. However he will not succeed in finding out minimal hypersurfaces expected to be in these families.

§ 1. The harmonic conjugate relation

For a subset $A \subset R^{n+1}$, we denote by $[A]$ the smallest linear subspace containing A in the following. For $A_1, A_2, \cdots, A_m \subset R^{n+1}$, let

$$[A_1,, A_2, \cdots, A_m] := [A_1 \cup A_2 \cup \cdots \cup A_m] .$$

Let S^n be the unit n-sphere in R^{n+1} given by

$$(1.1) \qquad \sum_{i=1}^{n+1} x_i^2 = 1$$