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A remark on a closed hypersurface with constant second mean curvature in a Riemann space

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Introduction. Y. Katsurada ([6][1], [5]) proved the following two theorems:

THEOREM A. *Let V^m be a closed orientable hypersurface in an Einstein space which admits a conformal Killing vector field ξ^i. If*

(i) *H_1 is constant,*

(ii) *$N_i\xi^i$ has fixed sign on V^m,*

then every point of V^m is umbilic, where H_1 and N_i denote the first mean curvature of V^m and the covariant component of the unit normal vector to V^m respectively.

THEOREM B. *Let V^m be a closed orientable hypersurface in a Riemann space of constant curvature which admits a conformal Killing vector field ξ^i. If*

(i) *H_ν is constant for a fixed ν $(2\leqq\nu\leqq m-1)$,*

(ii) *$k_1, k_2, \cdots, k_m$ are positive at each point on V^m,*

(iii) *$N_i\xi^i$ has fixed sign on V^m,*

then every point of V^m is umbilic, where k_α $(\alpha=1, 2, \cdots, m)$ and H_ν denote the principal curvature and the ν-th mean curvature of V^m respectively.

The present author [8] proved

THEOREM C. *Let V^m be a closed orientable hypersurface in a Riemann space which admits a conformal Killing vector field ξ^i. If*

(i) *H_2 is constant,*

(ii) *$k_1, k_2, \cdots, k_m$ are positive at each point on V^m,*

(iii) *$C^\alpha{}_{\beta;\alpha}$[2] $=0$ on V^m,*

(iv) *$N_i\xi^i$ has fixed sign on V^m,*

then every point of V^m is umbilic.

It is one of the interesting problems for us to find the conditions that

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1) Numbers in brackets refer to the references at the end of the paper.

2) $C_{\alpha\beta}$ are defined by $b_\gamma^\gamma b_{\alpha\beta}-b_\alpha{}^\gamma b_{\gamma\beta}$, where, $b_{\alpha\beta}$ and $g^{\alpha\beta}$ denoting the covariant component of the second fundamental tensor and the contravariant component of the metric tensor of V^m respectively, $b_\gamma^\gamma=b_{\alpha\beta}g^{\alpha\beta}$ and $b_\alpha{}^\gamma=b_{\alpha\beta}g^{\beta\gamma}$. And $C^\alpha{}_{\beta;\alpha}=C_{\alpha\beta;\gamma}g^{\alpha\gamma}$, where the symbol ";" means the covariant derivative.