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Higher order monodiffric difference equation

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1. Introduction. In [1] Berzsenyi has discussed the general solution of the first order monodiffric difference equation. Let D be the first quadrant of the discrete plane, and suppose $a\in D-\{0\}$ and $g\in M(D)$, then the general solution of the monodiffric equation $f'(z)-af(z)=g(z)$ with initial condition $f(0)=c$ is given by the monodiffric function

$$f(z)=cE(z, a)+\frac{1}{a}\left[g*E(z, a)\right] \qquad \text{for every } z\in D,$$

where $E(z, a)$ is the monodiffric exponential function (c. f. [2])

$$E(z, a)=(1+a)^{x}(1+ia)^{y} \qquad \text{for } z=x+iy .$$

In the sequel, we use the notation $e^{a,z}$ instead of $E(z, a)$.

In this paper, we extend Berzsenyi's results to more general cases, namely, the general solution of the n'th order monodiffric linear homogeneous difference equation

$$F^{(n)}(z)+c_{n-1}F^{(n-1)}(z)+\cdots+c_{1}F'(z)+c_{0}F(z)=0 , \tag{1. 1}$$

where the coefficients c_i $(i=0, 1, \cdots, n)$ are arbitrary constants.

2. Definition. Let $D=\{z|z=x+iy,\ x \text{ and } y \text{ are integers}\}$ and f be a complex-valued function defined on D. We define the monodiffric residue of f at z to be the value $Mf(z)$ given by

$$Mf(z)=(i-1)f(z)+f(z+i)-if(z+1). \tag{1. 2}$$

We say that f is monodiffric at z if $Mf(z)=0$. And a function which is monodiffric at every point in D is monodiffric on D. In this case, we write $f\in M(D)$. The monodiffric derivative f' of f is defined by

$$f'(z)=\frac{1}{2}\left[(i-1)f(z)+f(z+1)-if(z+i)\right].$$

We also use the symbol $\frac{df}{dz}$ or D_z.

3. The monodiffric exponential function $e^{a,z}$.

In [2], Isaacs has introduced the monodiffric exponential function $e^{a,z}$, it has a form $e^{a,z}=(1+a)^{x}(1+ia)^{y}$ for $z=x+iy$ and a is a complex number.