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Cluster sets on Riemann surfaces

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1. Introduction

In the theory of cluster sets of meromorphic functions defined in a domain in the z-plane, the next Theorem A and Theorem B are well-known. Let $w=f(z)$ be a meromorphic function in a domain D in the z-plane. Let b_0 be a point of boundary B of D and let E be a subset of B such that $b_0\in E$ and $b_0\in\overline{B-E}$ (the closure— is taken in the w-sphere). We denote by $C(f, b_0)$ the cluster set of $f(z)$ at b_0 and denote by $C_{B-E}(f, b_0)$ the boundary cluster set of $f(z)$ at b_0 modulo E. When D is the unit disk $\{z; |z|<1\}$, we denote by $C_{R-E}(f, b_0)$ the radial boundary cluster set of $f(z)$ at b_0 modulo E. Then there exists the next relation between the boundary $\partial C(f, b_0)$ of $C(f, b_0)$ and $C_{B-E}(f, b_0)$ (or $C_{R-E}(f, b_0)$) (cf. E. F. Collingwood and A. J. Lohwater [1] and K. Noshiro [7]).

THEOREM A. (*M. Tsuji* [10]) *If E is a compact set of capacity zero, then $\partial C(f, b_0)\subset C_{B-E}(f, b_0)$, that is $\Omega=C(f, b_0)-C_{B-E}(f, b_0)$ is an open set. And if $\Omega\neq\phi$, then every value of Ω is assumed infinitely often by $f(z)$ in any neighborhood of b_0 with the possible exception of a set of capacity zero.*

THEOREM B. (*M. Ohtsuka* [8]) *Let D be the unit disk. If E is a set of linear measure zero, then $\partial C(f, b_0)\subset C_{R-E}(f, b_0)$, that is $\Omega'=C(f, b_0)-C_{R-E}(f, b_0)$ is an open set. And if $\Omega'\neq\phi$, then every value of Ω' is assumed by $f(z)$ in any neighborhood of b_0 with the possible exception of a set of capacity zero.*

In this paper we study these theorems for meromorphic functions defined in an open Riemann surface. For this purpose, it is necessary to consider appropriate compactifications of Riemann surfaces. Z. Kuramochi [5] considered compactifications of Riemann surfaces with regular metrics and extended Theorem A to the case of Riemann surfaces (Theorem 1 in § 2). Our results in this paper are Theorem 2, Theorem 3 and Theorem 5 in § 2. Theorem 3 is an extention of Theorem B to the case of Riemann surfaces. We apply Kuramochi's method in [5] to prove these theorems.