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On a parametrix for the hyperbolic mixed problem with diffractive lateral boundary

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§ 1. Introduction and results.

Let Ω be a domain of the closed half space $\overline{R_{+}^{n+1}}=\{x;\ x=(x', x_n),\ x'=(x_0, \cdots, x_{n-1}),\ x_n\geqq 0\}$ containing a neighborhood of a point $x^{0'}=(0, x_1^0, \cdots, x_{n-1}^0)$ in its lateral boundary $\Gamma=\{x;\ x\in\Omega,\ x_n=0\}$ and let $P(x, D)$ be a differential operator of order 2 with C^∞ coefficients on $\bar{\Omega}$, which is normal hyperbolic with respect to x_0.

Now let us consider the mixed problem in Ω; for some $\delta>0$

$$(1.1)\qquad (P, B)\begin{cases} P(x, D)\,u=0 & \text{in } \Omega \text{ and for } x_n<\delta\,, \\ B(x, D)\,u=f & \text{in } \Gamma\,, \\ u=0 & x_0<0 \end{cases}$$

where $[0, \delta]\times\Gamma\subset\Omega$, the given boundary data f vanishes for $x_0<0$, $B(x, D)$ is a differential operator of order 1, and Γ is non-characteristic with respect to B.

Reweiting the principal symbol $P_2(x, \xi)$ of $P(x, D)$ in the following form:

$$(1.2)\qquad P_2(x, \xi)=\left(\xi_n-\lambda(x, \xi')\right)^2-\mu(x, \xi'),\ \xi=(\xi', \xi_n)\,,$$

we assume that Γ is diffractive *i. e.*, that for $(x, \xi)\in T_\Gamma^*(\Omega)$

$$(1.3)\qquad \left\{\xi_n-\lambda(x, \xi'),\ \mu(x, \xi')\right\}>0 \text{ when } \xi_n=\lambda(x, \xi') \text{ and } \mu(x, \xi')=0$$

where $\{f, g\}$ is the Poisson bracket and then such points $(x, \xi')\in T^*(\Gamma)$ with above properties are called diffractive ([8]).

Near a fixed diffractive point $(x^0, \xi^{0'})$, let $\lambda^\pm(x, \xi')$ be roots of $P_2(x^0, \xi', \xi_n)=0$ with respect to ξ_n such that for $\xi_0>0$

$$(1.4)\qquad \begin{aligned} &\lambda^\pm(x, \xi')=\lambda(x, \xi')\mp\sqrt{\zeta}\,\mu_3^{\frac{1}{2}}(x, \xi')\,, \\ &\mu_3(x, \xi')>0\,, \\ &\zeta=\xi_0-\mu_2(x, \xi'')\ \left(\xi''=(\xi_1, \cdots, \xi_{n-1})\right), \\ &\mu_2(x, \xi'') \text{ is real valued}\,, \\ &\sqrt{1}=1 \text{ and } \sqrt{-1}=-i\,. \end{aligned}$$