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A uniqueness theorem for holomorphic functions of exponential type

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§ 1. Introduction.

In this paper we treat a uniqueness theorem for holomorphic functions of exponential type on a half plane from the point of view of the theory of analytic functionals with non-compact carrier.

Avanissian and Gay [1] proved among others the following Theorem 1 using the theorey of analytic functionals of Martineau [4].

THEOREM 1. *Let $F(\zeta)$ be an entire function of type $<\pi$. If we have $F(-n)=0$ for every $n=1, 2, 3, \cdots$, then the entire function $F(\zeta)$ vanishes identically.*

Theorem 1 is a corollary to Carlson's theorem (see Boas [2] p. 153).

THEOREM 2. (Carlson) *Let $F(\zeta)$ be a holomorphic function on the half plane $\{\zeta=\xi+i\eta;\ \xi=\mathrm{Re}\,\zeta<0\}$. Suppose that there exist real numbers a, k with $0\leqslant k<\pi$ and $C\geqslant 0$ such that*

$$\left|F(\zeta)\right|\leqslant C\exp(a\xi+k|\eta|) \qquad \text{for } \mathrm{Re}\,\zeta<0\,.$$

If we have $F(-n)=0$ for $n=1, 2, 3, \cdots$, then the function $F(\zeta)$ vanishes identically.

We will prove Carlson's theorem by means of the theory of analytic functionals with non-compact carrier, which was introduced by the first named author [5] in connection with the theory of ultra-distributions of exponential growth of Sebastiaõ-è-Silva [6], namely Fourier ultra-hyperfunctions.

Following the sections we outline the results. In § 2 we define the fundamental space $\mathscr{Q}(L;k')$, the element of which is a holomorphic function in a tubular neighborhood L_ϵ of the closed half strip $L=[a, \infty)+i[k_1, k_2]$. A continuous linear functional on the space $\mathscr{Q}(L;k')$ is, by definition, an analytic functional with carrier in L and of (exponential) type $\leqslant k'$. The image of the Laplace transformation of $\mathscr{Q}'(L;k')$ is characterized in Theorem 3. As in Avanissian-Gay [1] we define in § 3 the transformation G_μ