Hokkaido Mathematical Journal Vol. 8 (1979) p. 1-22

On quasi Dirichlet bounded harmonic functions

By Zenjiro KURAMOCHI

(Received July 4, 1978)

In the present paper we denote by P, B, D, H, SPH and SBH, positive, bounded, Dirichlet bounded, harmonic, superharmonic and subharmonic respectively. Let R be a Riemann surface $\notin O_g$ and let $\{R_n\}: n=0, 1, 2, \cdots$, be an exhaustion with compact analytic relative boundary ∂R_n. We call a domain G a subdomain, if ∂G consists of at most an enumerably infinite number of analytic curves clustering nowhere in R. In this note we use simply a domain G in the meaning that G is a sum of subdomains G_i such that $\Sigma\partial G_i$ clusters nowhere in R. Let R^∞ be the universal covering surface of R and map R^∞ onto $|\zeta|<1$. In the previous[1] paper we proved.

Let $U(z)$ be an HD function in R. Then $U(z)$ is a harmonic function $U(\zeta)$ in $|\zeta|<1$, $\overline{\lim_{r\to1}}\int_0^{2\pi}|U(re^{i\theta})|^2\,d\theta<\infty$, $\zeta=re^{i\theta}$ and $U(\zeta)$ is representable by Poisson's integral. This is equivalent to $U(z)=U_1(z)-U_2(z)$, where $U_1(z)$ and $U_2(z)$ are positive *quasibounded harmonic* function (abbreviated by QHB).

The purpose of this paper is to extend the above theorem. Let G be a domain, if G is compact, we denote by H_g^G the solution of the Dirichlet problem with respect to the boundary value $g(z)$ on ∂G. If G is non compact and $g(z)\geqq 0$, we denote also by H_g^G the least positive $SPH\geqq g(z)$ on ∂G, *i. e.* $H_g^G=\lim_n H_{g_n}^{G\cap R_n}$, where $g_n(z)=g(z)$ on $\partial G\cap R_n$ and $g_n(z)=0$ on $\partial R_n\cap G$.

Let $G_2\subset G_1$ be domains. Let $w_{n,n+i}(z)$ be an HB in $G_1\cap R_{n+i}-G_2\cap(R_{n+i}-R_n)$ such that $w_{n,n+i}(z)=0$ on $\partial G_1\cap R_{n+i}+\partial R_{n+i}\cap(G_1-G_2)$, $=1$ on $(R_{n+i}-R_n)\cap G_2$. Then $w_{n,n+i}(z)\nearrow w_n(z)$ and $w_n(z)\downarrow: n\to\infty$. This limit is denote by $w(G_2\cap B, z, G_1)$ and is called $H. M.$[2] of $B\cap \overset{o}{G_2}$ relative to G_1. Let F be a closed set (or domain G). We denote $H_1^{CF}(H_1^{\overset{o}{C}G})$ by $w(F, z, R)$ $(w(G, z, R))$ simply. Let $U(z)$ be a positive SPH. If there exists no positive HB smaller than $U(z)$, we call $U(z)$ a *singular function.* If an HP is the limit of increasing sequence of HB functions, $U(z)$ is called *quasibounded harmonic function* (QHB). In this note we denote min $(M, U(z))$ by $U^M(z)$. Let $U(z)$ be a function (harmonic function). If