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A note on linearly compact modules

Dedicated to Professor Goro Azumaya on his 60th birthday

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Let R, S be rings[1)] and ${}_RM_S$ be an $R-S$-bimodule such that ${}_RM$ is linearly compact[2)] as left R-module. In this note we consider the conditions under which M_S, as right S-module, to be injective. Thus we have the following theorem which generalize Theoreme 2 in [1].

THEOREM. *Let ${}_RM_S$ be an R-S-bimodule such that ${}_RM$ is linearly compact. Then the following statements are equivalent:*

(1) *M_S is injective.*

(2) *M_S is absolutely pure, that is, every homomorphism of a finitely generated submodule of S_S^m to M_S is extended to that of S_S^m, where m is arbitaray natural number and S_S^m is a direct sum of m-fold copies of S_S.*

(3) *M_S is semi S-injective, that is, every homomorphism of a finitely generated right ideal of S to M_S is extended to that of S.*

(4) *${}_S\mathrm{Hom}_R(M, Q)$ is flat for every injevtive left R-module Q with essential socle.*

(5) *${}_S\mathrm{Hom}_R(M, K)$ is flat for every injective cogenerator K with essential socle.*

(6) *${}_S\mathrm{Hom}_R(M, K_0)$ is flat for some injective cogenerator K_0 with essential socle.*

In case where $S=\mathrm{End}({}_RM)$, the endomorphism ring of ${}_RM$, the above statements (1)$\sim$(6) *are equivalent also to*

(7) *${}_RM$ cogenerates the cokernel of every homomorphism ${}_RM^m \to {}_RM^n$, where m, n are arbitrary natural numbers. (Here one can set $m=1$).*

In order to prove the theorem we need the following

LEMMA[3)]. *Let A_S be a finitely generated right S-module, ${}_RM_S$ be an*

1) In what follows it is assumed that all rings have an identity element and all modules are unital.

2) A left R-module is called linearly compact if every finitely solvable system of congruences $x \equiv m_\alpha \pmod{M_\alpha}$, $\alpha \in \Lambda$, is solvable where $m_\alpha \in M$ and M_α are submodules of M.

3) Cf. [1], Lemma 2, also [4], Lemma 3.5.