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Finite groups admitting an automorphism of prime order I

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1. Introduction

Let G be a finite group and q a prime. We say that G is q-closed if G has a normal Sylow q-subgroup and q-nilpotent if G has a normal q-complement. In this paper we prove the following theorem.

THEOREM. *Let G be a finite group. Assume that G admits an automorphism α of order p, p a prime. Assume further that $C_G(\alpha)$ is a cyclic q-group for some odd prime q distinct from p. Then G is q-closed or q-nilpotent. In particular G is solvable.*

B. Rickman [8] prove the case $q \geqslant 5$, so we prove the case $q=3$.

2. Preliminaries

All groups considered in this paper are assumed finite. Our notation corresponds to that of Gorenstein [5].

(2.1) *Let A be a π'-group of automorphism of the π-group G, and suppose G or A is solvable. Then for each prime p in π, we have*

(1) *A leaves invariant some S_p-subgroup of G.*

(2) *Any two A-invariant S_p-subgroups of G are conjugate by an element of $C_G(A)$.*

(3) *Any A-invariant p-subgroup of G is contained in an A-invariant S_p-subgroup of G.*

(4) *If H is any A-invariant normal subgroup of G, then $C_{G/H}(A)$ is the image of $C_G(A)$ in G/H.*

(2.2) (Thompson)

A p-group P posseses a characteristic subgroup C with the following properties;

(1) *$cl(C) \leqslant 2$ and $C/Z(C)$ is elementary abelian.*

(2) $[P, C] \subseteq Z(C)$.

(3) $C_P(C) = Z(C)$.

(4) *Every nontrivial p'-automorphism of P induces a nontrivial automorphism of C.*