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Extension of involutions on spheres

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Introduction

Let Z_{2q} be a cyclic group of order $2q$ generated by T'. Suppose that a free involution T is given on the sphere S^n. If there exists a free Z_{2q}-action on S^n such that the restriction of the Z_{2q}-action to the Z_2-action coincides with T on S^n, i.e., $T'|Z_2=T'^q=T$, then we call that the involution T on S^n extends to a free Z_{2q}-action. In this paper, we show that:

THEOREM. *Let q be any integer and $n\geqq 1$. Then, every picewise linear (resp. topological) free involution on S^{2n+1} extends to a picewise linear (resp. topological) free Z_{2q}-action on S^{2n+1}.*

The theorem follows from a similar method to the proof of the following proposition.

PROPOSITION 3.1. *Let T be a free involution on a homotopy sphere Σ^{2n+1} such that the normal invariant $\eta(\Sigma^{2n+1}/T)\in Im\{p^*:[L^{2n+1}(2q), G/H]\to[p^{2n+1}, G/H]\}$ and $(q, |\Theta_{2n+1}(\partial\pi)|)=1$, where $p: p^{2n+1}\to L^{2n+1}(2q)$ is the projection and $H=O$, PL or TOP and $n\geqq 2$. Then, T extends to a free Z_{2q}-action on Σ^{2n+1}.*

§1 and §2 will be devoted to the preliminaries of the above proposition. In §3, we shall prove it and the above theorem.

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1. Definition of transfer

Let X^{2n-1} be a $(2n-1)$-dimensional closed oriented manifold with fundamental group π. Denote by $\mathscr{S}_H^{\varepsilon}(X)$ the set of ε-homotopy structures on X, where $H=O$ or PL and $\varepsilon=h$ or s. An ε-homotopy equivalence $f: M\to X$ determines a normal map

$$\begin{array}{ccc} \nu_M & \xrightarrow{b} & \xi \\ \downarrow & & \downarrow \\ M & \xrightarrow{f} & X, \end{array}$$