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# Notes on eigenvalues of Laplacian acting on $p$-forms

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## 1. Introduction.

By generalizing the method of Payne-Pólya-Weinberger ([4]), for a compact domain on a minimal hypersurface in the Euclidean space, S. Y. Cheng ([2]) proved an inequality between successive eigenvalues of the Laplacian acting on $C^{\infty}$-functions on this domain. On the other hand, M. Maeda ([3]) has got a similar result for a compact minimal submanifold with or without boundary in the unit sphere. The purpose of the present note is to present a similar inequality between successive eigenvalues of the Laplacian acting on $p$-forms (resp. 1-forms) on a compact and oriented minimal hypersurface (resp. minimal submanifold of any codimension) without boundary in the unit sphere.

Let $M$ be an $m(\geqslant 2)$-dimensional compact and oriented Riemannian manifold without boundary with the Riemannian metric $g$. For each $p=0, 1, \cdots, m$, $A^{p}(M)$ denotes the space of all differential $p$-forms on $M$. For $\omega$, $\eta\in A^{p}(M)$, we can define a $C^{\infty}$-function $(\omega|\eta)$ on $M$ as follows: $(\omega|\eta)$ is locally given by

$$(\omega|\eta)=\sum_{\substack{j_{1}\cdots j_{p}\\ k_{1}\cdots k_{p}=1}}^{m}\omega_{j_{1}\cdots j_{p}}\eta_{k_{1}\cdots k_{p}}g^{j_{1}k_{1}}\cdots g^{j_{p}k_{p}}$$

where we put $g_{jk}=g\left(\dfrac{\partial}{\partial z^{j}}, \dfrac{\partial}{\partial z^{k}}\right)$ for a local coordinate system $\{z^{j}; j=1, \cdots, m\}$ of $M$ and $(g^{jk})$ is the inverse matrix of $(g_{jk})$. The inner product $\langle\ ,\ \rangle$ on $A^{p}(M)$ is defined by $\langle\omega, \eta\rangle :=\int_{M}(\omega|\eta)\,dV_{M}$. Here $dV_{M}$ denotes the volume form of $M$. We put $\|\omega\|:=\sqrt{\langle\omega, \omega\rangle}$. Let $d: A^{p}(M)\rightarrow A^{p+1}(M)$ be the operator of exterior differentiation and $\delta: A^{p+1}(M)\rightarrow A^{p}(M)$ be the adjoint operator of $d$ with respect to $\langle\ ,\ \rangle$. Then the Laplace-Beltrami operator (Laplacian for short) $\Delta$ acting on $p$-forms is defined by $\Delta:=d\delta+\delta d: A^{p}(M)\rightarrow A^{p}(M)$. The Laplacian is elliptic and a self-adjoint operator. Thus all eigenvalues of the Laplacian form a discrete infinite sequence:

$$0=\lambda_{0}^{p}\leqslant\lambda_{1}^{p}\leqslant\cdots\leqslant\lambda_{n}^{p}\leqslant\cdots;\ \lambda_{n}^{p}\longrightarrow\infty$$

where each $\lambda_{n}^{p}$ is repeated as many time as its multiplicity. Let $\{\varphi_{r}; r=$