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On the Dirichlet problem for unbounded boundary functions

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1. We study the boundary behavior of the Dirichlet solution for an unbounded boundary function in this paper. We treat the Dirichlet problem by the Perron-Wiener-Brelot method. Let G be a bounded open set in the complex plane and let $f(b)$ be an extended real-valued function defined on its boundary ∂G. The upper class U_f^G for f is given by

$$U_f^G=\Big\{s\,;\ \textit{superharmonic, bounded below on } G\,,\ \varliminf_{z\to b} s(z)\geqq f(b)\ \textit{for all}\ b\in\partial G\Big\}$$

and set $\bar{H}_f^G(z)=\inf\{s(z);\ s\in U_f^G\}$ and $\underline{H}_f^G(z)=-\bar{H}_{-f}^G(z)$ for $z\in G$. If $\bar{H}_f^G=\underline{H}_f^G$ and both harmonic, then f is called to be resolutive and $H_f^G=\bar{H}_f^G=\underline{H}_f^G$ is called the Dirichlet solution for f. Any bounded continuous function on ∂G is resolutive. A point $b\in\partial G$ is called a regular boundary point if

$$\lim_{z\to b} H_g^G(z)=g(b) \tag{1}$$

for every bounded continuous function g on ∂G. Otherwise b is an irregular boundary point. The regularity is a local character, that is, if b is a regular boundary point for G and G_0 is an open subset of G for which b is also a boundary point, then b is regular for G_0.

Let $b\in\partial G$ be a regular boundary point. By M. Brelot's example [1], we know that (1) does not always hold for any unbounded function, even if it is continuous and resolutive. In this paper, we give a sufficient condition for (1) to hold. Our result is the following:

THEOREM. *Let f be an extended real-valued continuous and resolutive function on ∂G. Suppose $b_0\in\partial G$ is a regular boundary point. If there is a disk $B(b_0, r_0)=\{z;\ |z-b_0|<r_0\}$ such that the Dirichlet integral $D_{G\cap B(b_0,r_0)}(H_f^G)$ of H_f^G on $G\cap B(b_0, r_0)$ is finite, then $\lim_{z\to b_0} H_f^G(z)=f(b_0)$.*

For another sufficient condition, we refer to W. Ogawa [3].

2. **The proof of the theorem.** We refer to the monograph of Constantinescu-Cornea [2] for the definition and the properties of Dirichlet fun-