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Regularity of solutions to hyperbolic mixed problems with uniformly characteristic boundary

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§ 0. Introduction and results

Let G be a domain in $\boldsymbol{R}^n$ with smooth boundary ∂G. We consider the following mixed problem for a hermitian hyperbolic system P:

$$(P, B) \qquad \begin{cases} Pu=f & \text{in} \quad [t_1, t_2]\times G\,, \\ Bu=g & \text{on} \quad [t_1, t_2]\times \partial G\,, \\ u(t_1, x)=h & \text{for} \quad x\in G\,, \end{cases}$$

where $t_1<t_2$,

$$(0.1) \qquad P(t, x\,;\, D_t, D_x)=D_t+\sum_{j=1}^{n} A_j(t, x)\, D_j+C(t, x)\,,$$

A_j and C are $m\times m$ matrices, $A_j=A_j^*$, $D_t=-i\dfrac{\partial}{\partial t}$ and $D_j=-i\dfrac{\partial}{\partial x_j}$. The $B(t, x)$ is an $l\times m$ matrix of constant rank l and all of A_j, C and B are smooth and constant for large $|t|+|x|$.

For the sake of simplicity of description, throughout in the present paper we may suppose that G is the open half space $\{x_n>0\}$. Furthermore it is assumed that ∂G is uniformly characteristic for P, *i.e.*, the boundary matrix A_n is of constant rank d less than m near ∂G. This article is concerned with the L^2-well possedness for (P, B) and the regularity of solutions to (P, B) under the L^2-well posedness. In particular, we are here interested in the problem whose solution u satisfies the estimates of the type

$$(0.2) \qquad \begin{aligned} &\sum_{i=0}^{p}\Big\{e^{-\gamma t}\big|D_t^i u(t)\big|_{p-i,\gamma}^2+\gamma\int_{t_1}^{t} e^{-\gamma s}\big|D_s^i u(s)\big|_{p-i,\gamma}^2\, ds\Big\} \\ &\quad +\sum_{i+j\le p}\gamma\int_{t_1}^{t} e^{-\gamma s}\big\langle D_s^i D_n^j A_n u(s)\big\rangle_{p-\frac{1}{2}-i-j,\gamma}^2\, ds \\ &\le C_p\sum_{i=0}^{p}\Big\{e^{-\gamma t_1}\big|D_t^i u(t_1)\big|_{p-i,\gamma}^2 \\ &\quad +\gamma^{-1}\int_{t_1}^{t} e^{-\gamma s}\Big(\big|D_s^i f(s)\big|_{p-i,\gamma}^2+\big\langle D_s^i g(s)\big\rangle_{p+\frac{1}{2}-i,\gamma}^2\Big)\, ds\Big\}\,, \quad t_1<t<t_2\,, \end{aligned}$$