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The initial boundary value problem for inviscid barotropic fluid motion

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0. Introduction.

In his paper [2], Ebin showed the local in time existence of solutions to the initial boundary value problem for inviscid barotropic fluid motion in a bounded domain provided that the initial velocity is subsonic and the initial density is nearly constant. The purpose of the present article is to prove without the above assumptions the existence and continuous dependence of solutions for the data.

The inviscid barotropic fluid motion in a bounded domain $\Omega\subset\boldsymbol{R}^3$ with smooth boundary $\partial\Omega$ is governed by the standard equations of fluid mechanics;

$$(0.1)\qquad \left.\begin{aligned} &\frac{dv}{dt}+\frac{p'(\rho)}{\rho}\nabla\rho=K\\ &\frac{d\rho}{dt}+\rho\,\mathrm{div}\,v=0 \end{aligned}\right\} \quad \text{in } (0, T)\times\Omega .$$

Here d/dt denotes the material derivative,

$$\frac{d}{dt}=\frac{\partial}{\partial t}+v\cdot\nabla, \quad v\cdot\nabla=\sum_{j=1}^{3}v_j\frac{\partial}{\partial x_j}$$

and $v(t, x)=(v_1, v_2, v_3)$, $K(t, x)=(K_1, K_2, K_3)$, $\rho(t, x)$, $p(\rho(t, x))$ denote the velocity, the external force, the density, the pressure of fluid motion at time t and position $x\in\Omega$, respectively. For physical reasons we assume that ρ and the derivative $p'(\rho)$ in ρ are both positive. In addition to (0.1) we prescribe the initial data at $t=0$ and the boundary condition on $(0, T)\times\partial\Omega$;

$$(0.2)\qquad v(0)=v_0, \quad \rho(0)=\rho_0 \qquad \text{on } \Omega,$$

$$(0.3)\qquad \langle v, n\rangle=0 \qquad \text{on } (0, T)\times\partial\Omega$$

where $n=n(x)$ denotes the unit outerward normal to Ω at $x\in\partial\Omega$ and $\langle v, n\rangle$ or $v\cdot n$ denotes the standard inner product of v and n in $\boldsymbol{R}^3$.

The main result is