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On some exact sequences concerning with H-separable extensions

Dedicated to Prof. Kentaro Murata on his 60th birthday

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Introduction

In his paper [5] K. Hirata showed an exact sequence concerning with an H-separable extension A of B as follows

$$1 \longrightarrow \mathrm{Inn}(A|B) \longrightarrow \mathrm{Aut}(A|B) \longrightarrow P(A)$$

where $P(A)$ is the group of isomorphsm classes of some type of $A-A$-modules. But if we follow the same method as Azumaya algebras we can obtain also the following exact sequence

$$1 \longrightarrow \mathrm{Inn}(A|B) \longrightarrow \mathrm{Aut}(A|B) \longrightarrow \mathrm{Pic}(C)$$

where $\mathrm{Pic}(C)$ is the Picard group of the center C of A. Being stimulated by these facts the author tried to obtain some additional sequences. In this paper we will show that if A is an H-separable extension of B (*i.e.*, ${}_AA\otimes_B A_A \langle \oplus_A (A\oplus A\oplus\cdots\oplus A)_A)$ such that $V_A(B)\subset B$, there exists an exact sequence of group homomorphisms

$$1 \longrightarrow \mathrm{Inn}(A|B) \longrightarrow \mathrm{Aut}(A|B) \longrightarrow \mathrm{Pic}(C) \longrightarrow \mathrm{Pic}(B')$$

where $B'=V_A(V_A(B))$. From this we can induce an exact sequence

$$1 \longrightarrow \mathrm{Inn}(A|S) \longrightarrow \mathrm{Aut}(A|S) \longrightarrow \mathrm{Pic}(C) \longrightarrow \mathrm{Pic}(S)$$

in the case where A is an Azumaya C-algebra and S is a maximal commutative subring of A such that A is left S-projective, that is, A is an S/C-Azumaya algebra.

Sequence of group homomorphisms

Throughout this paper A is a ring with the identity 1 and B is a subring of A such that $1\in B$. $\mathrm{Aut}(A|B)$ denotes the group of all automorphisms of A which fix all elements of B and $\mathrm{Inn}(A|B)$ denotes the