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Mean curvature for certain p-planes in Sasakian manifolds

Dedicated to the memory of Professor Yoshie Katsurada

By Shigeyoshi FUJIMURA

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Introduction

Let (M, g) be an m-dimensional Riemannian manifold with a metric tensor g. We denote by $K(X, Y)$ the sectional curvature for a 2-plane spanned by tangent vectors X and Y at $x\in M$, and by π a p-plane at $x\in M$. Let $\{e_1, \cdots, e_m\}$ be an orthonormal base of the tangent space at $x\in M$ such that $\{e_1, \cdots, e_p\}$ spans π, which is called an adapted base for π. S. Tachibana [7] defined the mean curvature $\rho(\pi)$ for π by

$$\rho(\pi)=\frac{1}{p(m-p)}\sum_{a=1}^{p}\sum_{b=p+1}^{m}K(e_a, e_b),$$

which is independent of the choice of adapted bases for π, and proved the following:

THEOREM A (S. Tachibana, [7]). *In a Riemannian manifold (M, g) of dimension $m>2$, if the mean curvature for p-plane is independent of the choice of p-planes at each point, then*

(i) *for $p=1$ or $m-1$, (M, g) is an Einstein space,*

(ii) *for $2\leqq p\leqq m-2$ and $2p\neq m$, (M, g) is of constant curvature,*

(iii) *for $2p=m$, (M, g) is conformally flat.*

The converse is true.

Taking holomorphic $2q$-planes or antiholomorphic p-planes instead of p-planes, analogous results in Kählerian manifolds have been obtained.

THEOREM B (S. Tachibana [8] and S. Tanno [9]). *In a Kählerian manifold (M, g, J) of dimension $2n\geqq 4$, if the mean curvature for $2q$-plane is independent of the choice of holomorphic $2q$-planes at each point, then*

(i) *for $1\leqq q\leqq n-1$ and $2q\neq n$, (M, g, J) is of constant holomorphic sectional curvature,*

(ii) *for $2q=n$, the Bochner curvature tensor vanishes.*

The converse is true.

normal pair $\{X, Y\}$, then

$$K(X', Y')=K(X, Y)$$

for the 2-plane spanned by any $X'\in C(X)$ and $Y'\in C(Y)$.

PROOF. Putting $Y_1=\alpha Y+\beta\phi Y$ $(\alpha^2+\beta^2=1)$, since $\{X, Y_1\}$ is a ϕ-antiholomorphic orthonormal pair, we have

$$K(X, Y_1)=K(X, \phi Y_1),$$

from which it follows that