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On s-distance subsets in real hyperbolic space

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Abstract

It is shown that if X is an s-distance subset in real hyperbolic space H^{d}, then

$$|X|\leq\binom{d+s}{s}+\binom{d+s-1}{s-1}.$$

Introduction

A subset X in a metric space M is called an s-distance subset in M if there are s distinct distances $\alpha_{1}, \alpha_{2}, \cdots, \alpha_{s}$, and all the α_{i} are realized. Delsarte-Goethals-Seidel [6] have shown that the cardinality $|X|$ of an s-distance subset X in the d-dimensional unit sphere $S^{d}=\{(x_{1}, x_{2}, \cdots, x_{d+1})\,|\,x_{1}^{2}+x_{2}^{2}+\cdots+x_{d+1}^{2}=1\}\subset\boldsymbol{R}^{d+1}$ is bounded from above as

$$|X|\leq\binom{d+s}{s}+\binom{d+s-1}{s-1}. \tag{1}$$

Larman-Rogers-Seideal [9] and Bannai-Bannai [1] have shown that the same upper bound (1) is obtained for the cardinality of an s-distance subset in real Euclidean space $\boldsymbol{R}^{d}$. In this paper we prove that the same bound (1) is also true for an s-distance subset in the real hyperbolic space H^{d} of (topological) dimension d. That is :

THEOREM 1. *If X is an s-distance subset in H^{d}, then*

$$|X|\leq\binom{d+s}{s}+\binom{d+s-1}{s-1}.$$

1. PROOF OF THEOREM 1

The basic idea of the proof is the same as that of Delsarte-Goethals-Seidel [6] and Koornwinder [8]. Here we need a proper realization of the hyperbolic space H^{d} in $\boldsymbol{R}^{d+1}$.

(i) It is known that the hyperbolic space H^{d}, which is also called Lobatschewsky and Bolyai space, of dimension d is realized in a Euclidean space of $\boldsymbol{R}^{d+1}$ as