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Some multipliers on the space consisting of measures of analytic type

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§ 1 Introduction

Let G be a LCA group with the dual group $\hat{G}$. m_G denotes the Haar measure of G. Let $M(G)$ and $M_s(G)$ denote the space of all bounded regular (complex-valued) measures on G and the subspace of $M(G)$ consisting of all singular measures respectively. $L^1(G)$ denotes the usual group algebra, and $Trig(G)$ denotes the space of all trigonometric polynomials on G. $M_c(G)$ and $M_d(G)$ denote the subspaces of $M(G)$ consisting of continuous measures and discrete measures respectively. For a subset E of $\hat{G}$, $M_E(G)$ denotes the space consisting of measures in $M(G)$ whose Fourier-Stieltjes transforms vanish off E. For a subset E of $\hat{G}$, E^0 and E^- denote its interior and closure. '$\wedge$' and '$\vee$' denote the Fourier-Stieltjes transform and the inverse Fourier transform respectively. When there exists a nontivial continuous homomorphism from $\hat{G}$ into R (the reals), we shall say that a measure $\mu\in M(G)$ is of analytic type if $\hat{\mu}(\gamma)=0$ for $\gamma\in\hat{G}$ with $\psi(\gamma)<0$. We denote by $M^a(G)$ the set of measures in $M(G)$ which are of analytic type. For a subset B of $M(G)$, $B^\wedge$ means the set $\{\hat{\mu}\,;\,\mu\in B\}$. For $\mu\in M(G)$, we signify $\|\hat{\mu}\|$ by $\|\hat{\mu}\|=\|\mu\|$.

For a discrete measure $\nu\in M_d(G)$, $\mu*\nu$ belongs to $M_s(G)$ for every $\mu\in M_s(G)$. For a compact abelian group G, Doss proved that a multiplier on $M_s(G)$ is given by convolution with a discrete measure ([4]). In [7], Graham and MacLean obtained an analogous result for a LCA group. In section 2 of this paper, we prove the following:

THEOREM 2.3. *Suppose an ordering of $\hat{G}$ is given by nontrivial continuous homomorphism ψ from $\hat{G}$ into R. Let δ be a positive real number and Φ a multiplier on $L^1_{-\delta}(R)$ (the definition of $L^1_{-\delta}(R)$ will be stated in Definition 2.1). Then $\Phi\circ\psi$ is also a multiplier on $M^a(G)$ with the following properties:*

$$(\mathrm{I})\qquad S\big(M^a(G)\cap L^1(G)\big)\subset M^a(G)\cap L^1(G),$$

$$(\mathrm{II})\qquad S\big(M^a(G)\cap M_s(G)\big)\subset M^a(G)\cap M_s(G),$$