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An application of Evens' norm mapping

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1. Introduction

Let B_0 be the principal block of kG, where k is the prime field of characteristic $p>0$ and G is a finite group such that $G_p\neq 1$. G_p means a Sylow p-subgroup of G. All modules are finite dimensional vector spaces over k.

If a simple kG-module M does not belong to B_0, then $\oplus_{i=1}^{\infty} H^i(G, M)=0$. Therefore, if $\oplus_{i=1}^{\infty} H^i(G, M)\neq 0$ is proved for any simple kG-module M lying in B_0, then B_0 is written as $\{M|M$ represents an isomorphic class of simple kG-modules such that $\oplus_{i=1}^{\infty} H^i(G, M)\neq 0.\}$ (cf. Barnes, Schmid and Stammbach [1, § 3, Remark]). This characterization of B_0 is known, only when G is a p-nilpotent group (classical), a p-solvable group with an abelian Sylow p-subgroup [3, Theorem 2] or a metabelian group [3, Theorem 3].

The aim of this note is to prove the following Theorem 1 which generalizes [3, Theorem 2], by using Evens' norm mapping [2]. Specifically we show that B_0 is written as above, when G is a Frobenius group whose Frobenius kernel has the order divisible by p.

THEOREM 1. *Let G be a finite group with a normal p-subgroup D. Suppose M be a projective $k[G/D]$-module. We regard M as a kG-module. If $M^* =\mathrm{Hom}_k(M, k)$ is isomorphic to a kG-submodule of $S^i(\Omega_1(A)^*)$ for some normal abelian subgroup A of G such that $A\leq D$, then $H^{2qi}(G, M)\neq 0$. Here $\Omega_1(A)=\langle x\in A|x^p=1\rangle$, $q=|D:A|$ and $S=\oplus_{i=0}^{\infty} S^i$ is the symmetric algebra functor over k.*

[3, Theorem 2] is deduced from the case of $D=A$. Next we specialize to a Frobenius group and have the following.

THEOREM 2. *Let G be a Frobenius group with the Frobenius kernel N such that $N_p\neq 1$. Then $\oplus_{i=1}^{|H|} H^{2qi}(G, M)\neq 0$ for every simple kG-module M lying in B_0, where $q=|N_p : Z(N_p)|$, $Z(N_p)$ is the center of N_p and H is a Frobenius complement of G. Namely B_0 is described as the set $\{M|M$ represents an isomorphic class of simple kG-modules such that $\oplus_{i=1}^{\infty} H^i(G, M)\neq 0\}$.*