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A congruence between modular forms of half-integral weight

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Introduction

In [7], Shimura showed a natural correspondence between modular forms of integral weight and those of half-integral weight. On the other hand, primitive modular forms have congruences, as discussed by Doi-Ohta [1], which are closely connected with the special values of the zeta functions associated with these forms (Doi-Hida [2] and Hida [3], [4]). Thus it is natural to ask whether these congruences of primitive forms of integral weight induce the same congruences of the corresponding forms of half-integral weight. The purpose of this paper is to show an affirmative example to the following problem of Hida:

For primitive forms F, $G\in S(2k, 2N)$ with a congruence $F\equiv G \bmod \mathfrak{p}$, can one find corresponding eigenfunctions f, $g\in S((2k+1)/2, 4N)$ with $\mathfrak{p}$-integral Fourier coefficients such that $f\equiv g \bmod \mathfrak{p}$ and $f\not\equiv 0 \bmod \mathfrak{p}$?
Here, $\mathfrak{p}$ is a prime ideal of $\bar{\boldsymbol{Q}}$ and the congruence "$f\equiv g \bmod \mathfrak{p}$" means that all Fourier coefficients of $f-g$ vanish modulo $\mathfrak{p}$.

The converse statement is trivial, that is, the congruence $f\equiv g \bmod \mathfrak{p}$ and $f\not\equiv 0 \bmod \mathfrak{p}$ implies the congruence $F\equiv G \bmod \mathfrak{p}$ (see § 1).

We note here that the Fourier coefficients of the cusp form f are closely connected with the special values of a certain zeta function associated with F (Waldspurger [8] and Kohnen-Zagier [5]).

§ 1. The precise statement of the problem

For a positive integer N, put

$$\Gamma_0(N)=\left\{\begin{pmatrix} a & b \\ c & d\end{pmatrix}\in SL_2(\boldsymbol{Z})\,|\,c\equiv 0 \bmod N\right\},$$

$$\mathfrak{H}=\left\{z\in\boldsymbol{C}\,|\,\mathrm{Im}\,(z)>0\right\}.$$

Further we put