Hokkaido Mathematical Journal Vol. 12 (1983) p. 113-118

On a lifting problem of Fourier-Stieltjes transforms of measures

By Jyunji INOUE

(Received April 17, 1982; Revised October 13, 1982)

Let G and $\hat{G}$ be a LCA group and its dual group, respectively. $M(G)$ denotes the measure algebra on G, the Banach algebra of bounded regular complex Borel measures on G with convolution multiplication and total variation norm $\|\cdot\|$. $M_a(G)$ and $M_s(G)$ express the space of absolutely continuous measures and the space of singular measures on G with respect to the Haar measure of G, respectively. For $\mu\in M(G)$, $\hat{\mu}$ denotes the Fourier-Stieltjes transform of μ, and we put $B(\hat{G})=\{\hat{\mu}|\mu\in M(G)\}$, $A(\hat{G})=\{\hat{\mu}|\mu\in M_a(G)\}$, $B_s(\hat{G})=\{\hat{\mu}|\mu\in M_s(G)\}$. $B(\hat{G})$ is a Banach algebra with respect to the pointwise multiplication and the norm $\|\hat{\mu}\|=\|\mu\|$.

Let Λ be a closed subgroup of $\hat{G}$. The following theorem is well-known ([6]).

THEOREM 1. $B(\hat{G})|_{\Lambda}=B(\Lambda)$, $A(\hat{G})|_{\Lambda}=A(\Lambda)$.

It follows from theorem 1 that each member of $B(\Lambda)$ (resp. $A(\Lambda)$) can be lifted to a member of $B(\hat{G})$ (resp. $A(\hat{G})$), but it is not clear whether there exist any liftings which are linear maps from $B(\Lambda)$ to $B(\hat{G})$ (resp. from $A(\Lambda)$ to $A(\hat{G})$).

On the other hand, in the recent papers [4] and [5], we can find partial answers to this lifting problem.

THEOREM 2 (cf. [4] and [5]). *Let Λ be a discrete subgroup of $\hat{G}$, let H be the annihilator of Λ in G, and let W be a neighborhood of $0\in\hat{G}$. Choose a neighborhood U of $0\in\hat{G}$ and a probability measure $\rho\in M_a(G)$ such that* $\operatorname{supp}\hat{\rho}\subset U\subset W$ *and* $(U-U)\cap\Lambda=\{0\}$, *and put*

$$\hat{J}\hat{\mu}(\gamma)=\sum_{\alpha\in\Lambda}\hat{\mu}(\alpha)\,\hat{\rho}(\gamma-\alpha)\qquad(\hat{\mu}\in B(\Lambda),\ \gamma\in\hat{G}).$$

Then we have $\hat{J}\hat{\mu}\in B(\hat{G})$ with the following additional properties.

i) $\hat{J}\hat{\mu}|_{\Lambda}=\hat{\mu}$,

ii) $\|\hat{J}\hat{\mu}\|=\|\hat{\mu}\|$,

(*) iii) *$\hat{J}\hat{\mu}$ is positive definite if $\hat{\mu}$ is positive definite,*