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Some multipliers on the space consisting of measures of analytic type, II

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§ 0. Introduction

Let G be a LCA group with the dual group $\hat{G}$. m_G means the Haar measure of G. $M(G)$ and $L^1(G)$ denote the measure algebra and the group algebra respectively. Let $M_s(G)$ be the closed subspace of $M(G)$ consisting of the singular measures on G. For a subset E of $\hat{G}$, $M_E(G)$ denotes the space of measures in $M(G)$ whose Fourier-Stieltjes transforms vanish off E. We denote by E^0 and E^- the interior and closure of E respectively. "$\wedge$" and "$\vee$" denote the Fourier-Stieltjes transform and the inverse Fourier transform respectively. For a subset B of $M(G)$, $B^\wedge$ means a set $\{\hat{\mu}: \mu\in B\}$. Let R be the reals and $H^1(R)$ the Hardy space on R. Then, by the F. and M. Riesz theorem, $H^1(R)=\{\mu\in M(R): \hat{\mu}(x)=0 \text{ for } x<0\}$. When there is a nontrivial continuous homomorphism ψ: $\hat{G}\mapsto R$, we define $M^a(G)$ by $M^a(G)=\{\mu\in M(G): \hat{\mu}(\gamma)=0 \text{ for } \gamma\in\hat{G} \text{ with } \psi(\gamma)<0\}$. If $\mu\in M^a(G)$, we say that μ is a measure of analytic type.

For compact abelian groups G, Doss proved that each multiplier on $M_s(G)$ is given by convolution with a discrete measure on G ([3]). In [5], Graham and MaLean obtained an analogous result for LCA groups. On the other hand, the author in ([10], Theorem 2.3) proved that $\Phi\circ\psi$ becomes a multiplier on $M^a(G)$ for each multiplier Φ on $L^1_{-\delta}(R)$ $(\delta>0)$, where $L^1_{-\delta}(R)=\{f\in L^1(R): \hat{f}(x)=0 \text{ for } x<-\delta\}$. However it is natural to consider whether $\Phi\circ\psi$ becomes a multiplier on $M^a(G)$ for each multiplier Φ on $H^1(R)$ or not. There are two purpose in this paper. One is to prove that $\Phi\circ\psi$ becomes a multiplier on $M^a(G)$ for each multiplier Φ on $H^1(R)$. The other is to improve Theorem 2.4 in [10]. We state our results after the following definition.

DEFINITION 0.1 *Let E be a aubset of $\hat{G}$. A function Φ on $\hat{G}$ which is continuous on E^0 is called a multiplier (or multiplier function) on $M_E(G)$ if $\Phi\hat{\mu}\in M_E(G)^\wedge$ for each $\mu\in M_E(G)$. By the function Φ, there exists a unique bounded linear operator S on $M_E(G)$ such that $S(\mu)^\wedge=\Phi\hat{\mu}$. Thus defined S is called a multiplier operator (or merely multiplier) on $M_E(G)$*