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Complex powers of a class of pseudodifferential operators and their applications

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§ 0. Introduction

Seeley [15] has defined complex powers of elliptic operators P on a compact C^{∞} manifold Ω without boundary and examined asymptotic behaviors of the eigenvalues. For hypoelliptic operators satisfying, what is called, strong (H) condition of Hörmander [6], Kumano-go and Tsutsumi [9] have constructed complex powers suitable for them.

In the present paper we shall discuss complex powers $\{P_z\}_{z\in C}$ of a class of pseudodifferential operators P on the manifold Ω. Here the operator P has a symbol which vanishes exactly of order M on the characteristic set Σ, that is, P belongs to $OPL^{m,M}(\Omega;\Sigma)$ which is defined by Sjöstrand [16]. Then a condition of hypoellipticity of P with loss of $M/2$ derivatives is well known (see Boutet de Monvel [1], Boutet de Monvel-Grigis-Helffer [2] and Helffer [5]). Moreover, we shall develop asymptotic behaviors of the eigenvalues of P on the further hypotheses that P is self-adjoint and semibounded from below. For this purpose we have to construct two kinds of complex powers of P and use more convenient one for each situation.

For $M=2$, Menikoff-Sjöstrand [10], [11], [12], Sjöstrand [17] and Iwasaki [8] have studied asymptotic behaviors under various assumptions on Σ and P. In particular [12] and [17] have treated more general non-semibounded cases. Their methods are based on the construction of the heat kernel and an application of Karamata's Tauberian theorem. For general M, see also Mohamed [13]. However our method is essentially due to the theory of complex analysis (c. f. Smagin [18]). In order to carry out this, we shall study the first singularity of the trace of P_z. In elliptic case, Trace (P_z) has an extension to a meromorphic function in z in C with only simple poles ([15]). But in our case, even the first singularity is able to have a pole of second order. Accordingly we have to extend Ikehara's Tauberian theorem (see Wiener [19]).

The plan of this paper is as follows. In § 1 we give the precise definition of the operator mentioned above and a main theorem (Theorem 1.2).