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On the generalization of union of knots

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1. Introduction

In this note, we generalize the union of knots introduced by Kinoshita and Terasaka [1] and consider its relation to the problem on the primeness of knots with the unknotting number one. All of our treatments are done in the piecewise linear category. The author is grateful to all members of Topology seminars at Kobe University, Tsuda College and Hokkaido University.

1.1 First of all, we will define a set of knots that is the central concept of this note. We use the following usual notation: Let k be a knot in the 3-sphere S^3 and let C be a 3-cell in S^3 satisfying (*) and (**):

(*) k intersects with ∂C transversely in two points.

(**) For $C_0=cl(S^3-C)$, $(C_0, C_0\cap k)$ is the trivial cell pair.

Then we call $(C, C\cap k)$ the cell pair associated to the knot k. Now let K_1 and K_2 be knots in S^3. For non-negative integer n, $K_1+_nK_2$ denotes the set of knots constructed in the following manner: Let $\bar{K}$ denote the knot sum of K_1 and K_2, and S^2 be the decomposition sphere; $\bar{K}=K_1\underset{S^2}{\#}K_2$, and let $C_i(i=1, 2)$ be the 3-cell bounded by S^2 such that the cell pair $(C_i, C_i\cap\bar{K})$ is equivalent to the one associated to K_i. Let Γ be an arc in S^3 which satisfies the conditions (1) and (2):

(1) $\Gamma\cap\bar{K}=\partial\Gamma\cap\bar{K}=\{a, b\}$ (=two points) $\subset\bar{K}-S^2$ and $a\in C_1$,

(2) Γ and S^2 intersect transversely in n points.

We put $\Gamma\cap S^2=\{a_1, \cdots, a_n\}$, where the ordering is from a_1 to a_n counting from nearer point to a. Next we choose a regular n.b.d. of Γ, say $\bar{B}$, satisfying (3) and (4):

(3) Each component of $\bar{B}\cap S^2$ is a disc containing exactly one point of $\Gamma\cap S^2$.

Let D_0^i denote the disc in (3) containing a_i. Then $\bar{B}$ is decomposed into $(n+1)$ 3-cells by $D_0^1\cup\cdots\cup D_0^n$. Let $B_j(0\leq j\leq n)$ denote the j-th cell counting from the side of a. Then:

(4) $\bar{B}\cap\bar{K}=(B_0\cap\bar{K})\cup(B_n\cap\bar{K})$ and, $(B_0, B_0\cap\bar{K})$ and $(B_n, B_n\cap\bar{K})$ are trivial cell pairs.