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Indefinite Einstein hypersurfaces with nilpotent shape operators

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§ 1. Introduction

In [4], A. Fialkow classified Einstein hypersurfaces in indefinite space forms if the shape operator is diagonalizable. In [7], it was shown that if the shape operator A is not diagonalizable at each point then there are two possibilities: either $A^2=0$ or $A^2=-b^2I$, where b is a non-zero constant. In this paper those Einstein hypersurfaces with $A^2=0$ and rank A maximal are classified. The main results are the following.

2. 2 THEOREM. *If f: $M_n^{2n}\rightarrow N^{2n+1}(c)$ is an isometric immersion of M_n^{2n} into a space form of constant curvature c with $A^2=0$ and rank $A=n$, then the kernel of A is an integrable, totally isotropic and parallel n-dimensional distribution on M. (Here M has signature (n, n). This is a consequence of the conditions on A.)*

2. 3 COROLLARY. *If f is as above and $n>1$, then $c=0$.*

In Theorem 4. 2, isometric immersions f: $M_n^{2n}\rightarrow \boldsymbol{R}^{2n+1}$ with $A^2=0$ and rank $A=n$ are classified locally.

The Einstein hypersurfaces classified in Theorem 4. 2 provide a large family of examples of manifolds which have been studied extensively. A. G. Walker [10, 11, 12] and others (see [13], p. 278 for other references) investigated manifolds with parallel fields of planes. R. Rosca and others ([9], [1], [3]) study manifolds with spin-euclidean connections. In this case the spinor fields can be covariantly differentiated.

If f: $M_1^n\rightarrow N_1^{n+1}(c)$ is an isometric immersion with $A^2=0$ and rank $A=1$, then M_1^n also has constant sectional curvature c. L. Graves [5] classifies such f if $c=0$ and M is complete. In [6], Graves and Nomizu show that for $n\geqslant 4$ there are no umbilic-free isometric imbeddings from $S_1^n(1)$ into $S_1^{n+1}(1)$.