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A note on an isometric imbedding of upper half-space into the anti de Sitter space

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Introduction. K. Nomizu [3] studied the upper half-space $U_n=\{(x_1, \cdots, x_n)\,;\ x_n>0,\ x_1, \cdots, x_{n-1}\in \boldsymbol{R}\}$ with the Lorentz metric

$$(1)\qquad ds_0^2=(dx_1^2+\cdots+dx_{n-1}^2-dx_n^2)/x_n^2$$

which has constant sectional curvature 1. U_n is diffeomorphic to the matrix group G_n consisting of all $n\times n$ matrices of the form

$$g=\begin{bmatrix} x_n & & x_1 \\ & \ddots & \\ & x_n & x_{n-1} \\ 0 & 0 & 1 \end{bmatrix},\ \text{where } x_n>0,\ x_1, \cdots, x_{n-1}\in \boldsymbol{R}$$

by

$$g\in G_n \longrightarrow (x_1, \cdots, x_{n-1}, x_n)\in U_n\,.$$

The group G_n is of type $\mathfrak{S}$ in the sense of [2] and it admits a left-invariant Lorentz metric with any prescribed constant k as its constant sectional curvature (Theorem 1, [2]). The left translations on G_n

$$\begin{bmatrix} x_n & & x_1 \\ & \ddots & \vdots \\ & x_n & x_{n-1} \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} a & & b_1 \\ & \ddots & \vdots \\ & a & b_{n-1} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_n & & x_1 \\ & \ddots & \vdots \\ & x_n & x_{n-1} \\ 0 & 0 & 1 \end{bmatrix}$$

correspond to the action of G_n on U_n by

$$(2)\qquad (x_1, \cdots, x_{n-1}, x_n) \longrightarrow (ax_1+b_1, \cdots, ax_{n-1}+b_{n-1}, ax_n)\,.$$

The Lorentz metric (1) on U_n is invariant by the action (2) of G_n and corresponds to a left-invariant Lorentz metric on the group G_n of constant sectional curvature 1.

In this note, we shall consider the upper half-space U_n with the Lorentz metric

$$(3)\qquad ds^2=(-dx_1^2+dx_2^2+\cdots+dx_n^2)/x_n^2$$

which corresponds to a left invariant Lorentz metric on G_n of constant