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A generalization of monodiffric Volterra integral equations

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1. Introduction

Various different types of discrete Volterra integral equations have been discussed by Deeter [2], Duffin and Duris [3], Fenyes and Kosik [4], and Tu [6, 7]. In [4], Fenyes and Kosik have solved discrete Volterra equations of the type

$$nf_n+\sum_{k=0}^{n} f_k g_{n-k}=h_n$$

by the method of operational calculus. By using the convolution product for discrete function theory, Duffin and Duris [3] discussed a solution of the discrete Volterra type

$$u(z)=f(z)+\lambda\int_0^z k(z-t): u(t)\ \mathrm{dt}, \text{ where } \lambda \text{ is a constant.} \tag{1.1}$$

On the other hand, Deeter [2] gave a different approach to the equation (1.1) by using some further results of operational calculus. Our aim in this paper is to define the convolution product of p-monodiffric functions and to prove some properties of p-monodiffric functions. We then find the general solutions of the generalized monodiffric Volterra type integral equations (1.1). When $p=1$, our results reduce to the classical results of p-monodiffric functions which have been developed by Berzsenyi [1] and Tu [6].

2. Definitions and Notations

Most of the definitions and notations given here are taken from reference [7]. Let $\boldsymbol{C}$ be the complex plane,
$D=\{z \in \boldsymbol{C} \mid z=x+iy\}$ where $x, y \in\{pj \mid j=0,1,2, \cdots, 0<p\leqq 1\}$ and $f: D\rightarrow\boldsymbol{C}$.

DEFINITION 1. The p monodiffric residue of f at z is the value

$$M_pf(z)=(i-1)f(z)+f(z+ip)-if(z+p). \tag{2.1}$$

DEFINITION 2. The function f is said to be p monodiffric at z if $M_pf(z)=0$. The function f is said to be p monodiffric in D if it is p monodiffric at any point in D (denoted by $f\in M_p(D)$).

DEFINITION 3. The p monodiffric derivative f' of f is defined by