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Systems of equations of hyperbolic-parabolic type with applications to the discrete Boltzmann equation

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§ 1. Introduction

In this paper we continue the study of linear symmetric systems of the form

$$A^{0}w_{t}+\sum_{j=1}^{n}A^{j}w_{x_{j}}-\sum_{j,k=1}^{n}B^{jk}w_{x_{j}x_{k}}+Lw=0, \tag{1.1}$$

where $t\geq 0$, $x=(x_{1}, \cdots, x_{n})\in \boldsymbol{R}^{n}$ and w is a function of the variables t and x, valued in $\boldsymbol{R}^{m}$. A^{0}, $A^{j}(j=1, \cdots, n)$, $B^{jk}=B^{kj}(j, k=1, \cdots, n)$ and L are $m\times m$ constant matrices. For notational convenience, we set

$$\begin{aligned} A(\omega)&=\sum_{j=1}^{n}A^{j}\omega_{j},\\ B(\omega)&=\sum_{j,k=1}^{n}B^{jk}\omega_{j}\omega_{k}, \end{aligned} \tag{1.2}$$

where $\omega=(\omega_{1}, \cdots, \omega_{n})$ is a unit vector in $\boldsymbol{R}^{n}$. The first assumptions on the coefficient matrices can be stated as follows.

CONDITION 1.1. (i) $A^{j}(j=1, \cdots, n)$ and $B^{jk}(j, k=1, \cdots, n)$ are real symmetric matrices and for each $\omega\in S^{n-1}$, $B(\omega)$ is nonnegative definite.

(ii) A^{0} and L are real symmetric matrices. Furthermore, A^{0} is positive definite and L is nonnegative definite.

The above condition gives a stable nature to the system (1.1) but it is not strong enough to guarantee the decay of solutions. We look for nontrivial solutions of the linear homogeneous equation

$$\lambda A^{0}\phi+\{L+\xi A(\omega)-\xi^{2}B(\omega)\}\phi=0, \tag{1.3}$$

for $\xi\in i\boldsymbol{R}$ and $\omega\in S^{n-1}$. The admissible values of λ are the zeros of $\det(\lambda A^{0}+L+\xi A(\omega)-\xi^{2}B(\omega))$. We write $\lambda=\lambda(\xi, \omega)$ and define what we call the strict dissipativity.

DEFINITION 1.1. The system (1.1) is said to be strictly dissipative, if the real part of $\lambda(\xi, \omega)$ is negative for each $\xi\in i\boldsymbol{R}\backslash\{0\}$ and $\omega\in S^{n-1}$.

The main purpose of the present paper is to prove that the strict dissipativity brings about the decay of solutions. The result seems to be new, because the rotational invariance is not assumed to hold for (1.1). We note that, in the previous works ([8], [6]), the decay estimates were obtained under CONDITION 1.1 and an additional condition which is as