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On the number of irreducible characters in a finite group II

Dedicated to Professor Hirosi Nagao on his 60th birthday

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§ 1 Introduction.

Let F be an algebraically closed field of characteristic p, and G be a finite group with a Sylow p-subgroup P. Let B be a block ideal of the group algebra FG which can be regarded as an indecomposable direct summand of FG as an $F(G\times G)$-module. We denote by $k(B)$ and $l(B)$ the number of irreducible ordinary and modular characters in B, respectively. In [8] the author introduced two invariants $m(B)$ and $n(B)$ associated with B that is the number of indecomposable direct summands of $B_{\Delta(P)}$ and $B_{P\times P}$, where Δ is the diagonal map from G to $G\times G$. We obtained some relations among four invariants $k(B)$, $l(B)$, $m(B)$ and $n(B)$, and it turned out that relation between $m(B)$ and $n(B)$ has a strong resemblance to that between $k(B)$ and $l(B)$. Furthermore, in [9] we proved that $l(B)\leqq n(B)$ and investigate the structure of B when equality holds. In this paper we will show that $|P:D|k(B)\leqq m(B)$ if a defect group D of B is contained in the center of P.

Let us set $|P|=p^{a}$, $|D|=p^{d}$ and $\dim_{F}B=p^{2a-d}v(B)$, where $v(B)=u(B)^{2}w(B)$ is the p'-number mentioned in [2] and [8]. Then our results can be written as the following diagram,