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A Class of Functions Defined by Using Hadamard Product

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Abstract

We introduce a class $\boldsymbol{P}_{\alpha}[\beta, \gamma]$ of functions defined by using Hadamard product $f*S_{\alpha}(z)$ of $f(z)$ and $S_{\alpha}(z)=z/(1-z)^{2(1-\alpha)}$. The object of the present paper is to determine extreme points, coefficient inequalities, distortion theorems, and radii of starlikeness and convexity for functions in $\boldsymbol{P}_{\alpha}[\beta, \gamma]$. Further, we give distortion theorems for fractional calculus of functions belonging to the class $\boldsymbol{P}_{\alpha}[\beta, \gamma]$.

1. Introduction

Let $\boldsymbol{A}$ denote the class of functions of the form

$$(1.1)\qquad f(z)=z+\sum_{n=2}^{\infty}a_{n}z^{n}$$

which are analytic in the unit disk $\boldsymbol{U}=\{z:|z|<1\}$. And let $\boldsymbol{S}$ denote the subclass of $\boldsymbol{A}$ consisting of analytic and univalent functions $f(z)$ in the unit disk $\boldsymbol{U}$. A function $f(z)$ in $\boldsymbol{S}$ is said to be starlike of order α if

$$(1.2)\qquad \operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\}>\alpha \qquad (z\in\boldsymbol{U})$$

for some $\alpha(0\leqq\alpha<1)$. We denote by $\boldsymbol{S}^{*}(\alpha)$ the class of all starlike functions of order α. Further, a function $f(z)$ in $\boldsymbol{S}$ is said to be convex of order α if

$$(1.3)\qquad \operatorname{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\}>\alpha \qquad (z\in\boldsymbol{U})$$

for some $\alpha(0\leqq\alpha<1)$. And we denote by $\boldsymbol{K}(\alpha)$ the class of all convex functions of order α. It is well-known that $f(z)\in\boldsymbol{K}(\alpha)$ if and only if $zf'(z)\in\boldsymbol{S}^{*}(\alpha)$, and that $\boldsymbol{S}^{*}(\alpha)\subseteq\boldsymbol{S}^{*}(0)\equiv\boldsymbol{S}^{*}$, and $\boldsymbol{K}(\alpha)\subseteq\boldsymbol{K}(0)\equiv\boldsymbol{K}$ for $0\leqq\alpha<1$.

These classes $\boldsymbol{S}^{*}(\alpha)$ and $\boldsymbol{K}(\alpha)$ were first introduced by Rebertson [9],

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