Hokkaido Mathematical Journal Vol. 15 (1986), p. 21~46

Infinitesimal Deformations of Cusp Singularities

To the memory of Professor Takeshi Onodera

By Iku NAKAMURA

(Received October 30, 1984)

Introduction. The purpose of this article is to compute infinitesimal deformations $\boldsymbol{T}^1$ of cusp singularities of two dimension. It is known that cusp singularities of three or higher dimension are rigid [3]. Let T be a cusp singularity of two dimension. The minimal resolution of T has a cycle of rational curves as its exceptional set. Let C be the (reduced) cycle of rational curves, r the number of irreducible components of C. Then our main consequence is that $\dim \boldsymbol{T}^1$ is equal to $r-C^2$ (if $C^2 \leqq -5$). This solves a conjecture of Behnke [1] in the affirmative. It should be mentioned that he himself solved his this conjecture in [2] in a manner different from ours.

Our method is just the same as in [1], where Behnke showed that $\boldsymbol{T}^1$ is the space of solutions of a system of infinitely many linear equations in infinitely many variables. We reduce this to finitely many linear equations in finitely many variables (§ 2). After preparing some lemmas about support points (§ 3), we solve the system of finitely many linear equations in the case $C^2 \leqq -5$ (§ 4). The same method yields a complete description of $\boldsymbol{T}^1$ in the case $-1 \geqq C^2 \geqq -4$ except for $r=-C^2=1$. The consequence leads us to a conjecture about deformations of $T_{p,q,r}$ and $\Pi_{p,q,r,s}$ in the cases $C^2=-3,\ -4$ (§ 5).

Contents.

Index of notations and terminologies