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LOCAL TOPOLOGICAL MODELS OF ENVELOPES

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1. Introduction

Let X and Y be smooth equidimensional manifolds and let $\Gamma \subset X \times Y$ be a smooth hypersurface with the natural projection $\pi_X : \Gamma \rightarrow X$, $\pi_Y : \Gamma \rightarrow Y$ submersive. Given $x \in X$ (resp. $y \in Y$) we denote by Γ_x (resp. Γ_y) the smooth submanifold $\pi_X^{-1}(x)$ (resp. $\pi_Y^{-1}(y)$) which we can think of as a smooth hypersurface in Y (resp. X). If $M \subset X$ is a smooth submanifold we can form the envelope $E(M)$ of the Γ_x in Y, for $x \in M$. In [2], Bruce discussed local models for $E(M)$. He has shhown that if dim $Y \leqq 6$ then the envelope $E(M)$ has generic Legendrian singularities for a residual set of embeddings $M \rightarrow X$. The stratified equivalence theory will be needed when dim $Y \geqq 7$. But he has remarked that his set up does not connect well with Looijenga's canonical stratification discussed in [8].

In this paper we shall avoid the difficulty by using a modification of Mather's stratification. The main result is the following.

THEOREM (1.1). *For a residual set of embeddings $M \rightarrow X$, local pictures of the envelope $E(M)$ are given by critical values of MT-stable map germs. Here, we call a map germ MT-stable if it is transverse to the canonical stratification of a jet space which is introduced in* ([5], [7]).

Of course, the critical value of a MT-stable map germ has the canonical Whitney stratification. Hence, we have a finite number of local models of generic envelopes up to stratified equivalence. In [4], it is proved that generic Legendrian singularities are singularities of MT-stable map germs. Hence, singularities of the generic envelopes and generic Legendrian singularities are in the same class of singularities of smooth mappings. Examples of such envelopes are given in [2].

All map germs and diffeomorphisrs considered here, are differentiable of class C^∞, unless stated otherwise.

2. Formulations (Including a quick reviews of Bruce [2])

In this section we introduce the definition of $E(M)$ and fundamental