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On the energy decay of a weak solution of the M. H. D. equations in a three-dimensional exterior domain

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Introduction

Let O be a bounded domain in $\boldsymbol{R}^3$ with smooth boundary ∂O. We set $\Omega=\boldsymbol{R}^3-O$. For simplicity, we assume that Ω is simply connected. In $Q:=\Omega\times(0,\infty)$, we consider the following magnetohydrodynamic (M. H. D.) equations;

$$\text{(M. H. D.)}\quad\begin{array}{ll} \partial_t u-\Delta u+(u,\nabla)u+B\times\mathrm{rot}B+\nabla\pi=f & \text{in } Q,\\ \partial_t B-\Delta B+(u,\nabla)B-(B,\nabla)u=0 & \text{in } Q,\\ \mathrm{div}\ u=0,\ \mathrm{div}\ B=0 & \text{in } Q,\\ u=0,\ B\cdot\nu=0,\ \mathrm{rot}B\times\nu=0, & \text{on } \partial\Omega\times(0,\infty),\\ u|_{t=0}=u_0,\ B|_{t=0}=B_0. & \end{array}$$

Here $u=u(x,t)=(u^1(x,t), u^2(x,t), u^3(x,t))$, $B=B(x,t)=(B^1(x,t), B^2(x,t), B^3(x,t))$ and $\pi=\pi(x,t)$ denote respectively the unknown velocity field of the fluid, magnetic field and pressure of the fluid, $f=f(x,t)=(f^1(x,t), f^2(x,t), f^3(x,t))$ denotes the given external force, $u_0=u_0(x)=(u_0^1(x), u_0^2(x), u_0^3(x))$ and $B_0=B_0(x)=(B_0^1(x), B_0^2(x), B_0^3(x))$ denote the given initial data and ν denotes the unit outward normal on $\partial\Omega$.

Our problem reads as follows.

PROBLEM

Construct a weak solution $\{u, B\}$ *of* (M. H. D.) *on* $(0,\infty)$ *such that*

$$E(t):=(1/2)\int_\Omega(|u(x,t)|^2+|B(x,t)|^2)\,dx$$

tends to zero as $t\to\infty$.

In this paper, we solve this problem affimatively. To this end, we shall use the methods developed by Masuda [5] and Sohr [10] in the case of the Navier-Stokes equations.

As is shown by Masuda [5, Corollary 2], we shall show at first that if $\{u, B\}$ is a weak solution of (M. H. D.) such that $E(t)$ tends to some constant E as $t\to\infty$, then $E=0$. For such a weak solution, we shall