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# A note on a theorem of Wada

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In his papers [6], [7], [8] T. Wada introduced and studied two new invariants, which may be associated with a $p$-block $B$ of a finite group $G$. If $B$ is considered as an ideal in the group algebra $FG$, where $F$ is an algebraically closed field of characteristic $p>0$ and $P$ is a $p$-Sylow subgroup of $G$, then $m(B)$ (or $n(B)$) is the number of indecomposable summands of $B$ when restricted to the diagonal group $\Delta(P)$ (or $P\times P$). A main result of [8] was that if $D=\delta(B)$ is a defect group of $B$ and if $D\leq Z(P)$, then

(*) $\quad |P:D|k(B)\leq m(B)$,

where $k(B)$ is the number of ordinary characters in $B$. Some examples were given to show that (*) does not hold in general. In this note we prove a result which has as a consequence the following obvious generalization of Wada's theorem:

Let $B$ be a $p$-block with $\delta(B)=D$. If $P_1$ is a $p$-Sylow subgroup of $DC_G(D)$ then

$$|P_1:D|k(B)\leq m(B).$$

By Brauer's second main theorem on blocks $k(B)$ may be decomposed according to the subsections of $B$ (See [4], IV, § 6). To prove our main result we decompose $m(B)$ in a similar way and compare the corresponding summands of $k(B)$ and $m(B)$.

Let $P$ be a $p$-Sylow subgroup of $G$ and $B$ a block with $\delta(B)=D$. Then there is an integer $v(B)$ such that

(1) $\quad \dim_F B=p^{2a-d}v(B)$

(see [2] or [8]). A subsection for $G$ is a pair $(\pi, b_\pi)$ where $\pi$ is a $p$-element in $G$ and $b_\pi$ a block of $C_G(\pi)$. If $b_\pi^G=B$, we call $(\pi, b_\pi)$ a $B$-subsection.

If $(\pi, b_\pi)$ is a subsection, define

(2) $\quad m(\pi, b_\pi):=\dfrac{1}{|P|}|\pi^G\cap P|\dim b_\pi.$

(here $\pi^G$ is the $G$-conjugacy class containing $\pi$).