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On some properties of γ_p-summing operators in Banach spaces

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§ 1. Introduction

The aim of this paper is to give new characterizations of Banach spaces which are of stable type p and of SQ_p type (or of S_p type) by the properties of γ_p-summing operators $(0<p\leqq 2)$.

The γ_p-summing operator was introduced by Thang and Tien [23]. Let E, F be Banach spaces and $0<p\leqq 2$. Denote by $L(E, F)$ the set of all bounded linear operators from E into F. Then an operator T in $L(E, F)$ is γ_p-summing if for each sequence $\{x_n\}\subset E$ with $\sum_n |<x_n, x'>|^p<\infty$ for all $x'\in E'$, the series $\sum_n T(x_n)\theta_n^{(p)}$ converges almost surely (a. s.) in F, where $\{\theta_n^{(p)}\}$ is a sequence of independent identically distributed real random variables with the characteristic function (ch. f.) $\exp(-|t|^p)$, $t\in R$. Denote by $\Pi_{\gamma_p}(E, F)$ the set of all γ_p-summing operators from E into F.

The γ_p-Radonifying operator was introduced by Linde, Mandrekar and Weron [6] as follows. Let p be $1<p\leqq 2$ and $1/p+1/p'=1$. Then an operator T in $L(L_{p'}, F)$ is γ_p-Radonifying if $\exp(-\|T'(x')\|^p)$, $x'\in F'$, is the ch. f. of a Radon measure on F, where T' is the adjoint of T. Denote by $\sum_p(L_{p'}, F)$ the set of all γ_p-Radonifying operators from $L_{p'}$ into F.

The classification problem of type p-stable Banach spaces, $1<p\leqq 2$, was studied by Chobanjan and Tarieladze [1], Kwapien [4], Linde, Mandrekar and Weron [6], Linde, Tarieladze and Chobanjan [7], Mandrekar and Weron [12], Mouchtari [15] and Thang and Tien [23]. The classifications by these authors are all based upon the properties of γ_p-Radonifying operators. In this paper, we shall adopt the γ_p-summing operators for such classifications. The reason why we use the γ_p-summing operators instead of γ_p-Radonifying operators will become clear through the discussions in Sections 3 and 4. Here we point out that $\Pi_{\gamma_p}(E, F)$ has the so-called ideal property, that is, if $T\in\Pi_{\gamma_p}(E, F)$, then $TS\in\Pi_{\gamma_p}(G, F)$ for every Banach space G and every $S\in L(G, E)$. But in general, for $1<p<2$, $\sum_p(L_{p'}, F)$ does not have the ideal property even in the case $G=L_{p'}$. In the classification of type p-stable Banach spaces, $1<p\leqq 2$, Mandrekar and Weron