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Solvable Generation of Finite Groups

To Bertram Huppert on his sixtieth birthday

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1. Introduction In their paper [1], Aschbacher and Guralnick proved that any finite group G is generated by a pair of conjugate solvable subgroups. The purpose of this note is to show that we can impose some conditions on how the generating subgroups are embedded in G. More precisely, we will prove the following theorem.

THEOREM *Let G be any finite group. Then, there is a solvable subgroup S such that*

(1) $G=<S, S^{g}>$ *for some element g of G,*

(2) *the conjugacy class of the subgroup S is stable under the group* Aut G *of automorphisms of G, and*

(3) $N_{G}(S)=S$.

In this note, a subgroup which satisfies the second condition will be called a (*)-subgroup of G. Thus, a subgroup H is a (*)-subgroup of G if, for any automorphism σ of G, there is an element x, depending on σ, such that

$$\sigma(H)=x^{-1}Hx.$$

The conditions (2) and (3) impose some restrictions on the way the subgroup S is embedded in G. Since every maximal solvable subgroup of any finite group is self-normalizing, to impose the condition (3) alone is trivial, but to put the two conditions (2) and (3) together on S seems to be not so trivial. It may be possible to impose further conditions on the embedding of S or on the properties of the element g.

We add the following remarks. Let G be any finite group. Then, a conjugacy class of solvable subgroups which satisfies the conditions (1), (2), and (3) is not necessarily unique. It follows from elementary group theory ([2], p. 99) that the normalizer of an S_{p}-subgroup is a self-normalizing (*) -subgroup. In particular, let H be the normalizer of an S_{2}-subgroup of G. (If the order $|G|$ is odd, we have $H=G$.) By the Feit-Thompson Theorem, H is solvable. It is fairly obvious that the group G is generated by *all the conjugates of* H, and that there are groups G in which any pair of conjugates of H generates a proper subgroup of G.