Hokkaido Mathematical Journal Vol. 17 (1988), p. 345-353

Generalized variation and translation operator in some sequence spaces

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(Received August 5, 1987, Revised February 23, 1988)

Abstract. There are defined and investigated some spaces of sequences provided with two-modular structure given by generalized variations and the translation operator. The results are applied to obtain an approximation theorem by means of translated sequences.

1. Let $x=(t_i)=(t_i)_{i=0}^{\infty}$ be a sequence of real numbers. We denote also $(x)_j=t_j$ for $j=0, 1, 2, \ldots$. We introduce two auxiliary notations: this of the Φ-variation of x and that of the sequential modulus of x.

1.1. Let X be the space of all real sequences and let Φ be a φ-function (see e.g. [4], 1.9). The Φ-variation $w_{\Phi}(x)$ of $x\in X$ is defined as

$$w_{\Phi}(x)=\sup_{(n_i)}\sum_{i=1}^{\infty}\Phi(|t_{n_i}-t_{n_{i-1}}|),$$

where the supremum runs through all increasing subsequences (n_i) of indices (see [2]). w_{Φ} is a pseudomodular in X defining the modular space

$$X_{\Phi}=X_{w_{\Phi}}=\{x\in X : w_{\Phi}(\lambda x)\longrightarrow 0 \text{ as } \lambda\longrightarrow 0_+\}$$

(see [7], [5] and also [8]). $\|\cdot\|_{\Phi}$ will denote the Luxemburg pseudonorm in X_{Φ} (see [4]). It is easily seen that $X_{\Phi}\subset c$, where c is the space of convergent sequences, and X_{Φ} is strongly modular complete and complete in the norm (see [2] and [5]).

1.2. Given any sequence $x=(t_i)_{i=0}^{\infty}$, we write

$$(\tau_m x)_j=\begin{cases} t_j & \text{for } j<m, \\ t_{m+j} & \text{for } j\geq m, \end{cases}$$

where $m, j=0, 1, 2, \ldots$ (see [3], also [4], 7.17). The sequence $\tau_m x=((\tau_m x)_j)_{j=0}^{\infty}$ is called the m-translation of the sequence x.

1.3. The sequential modulus of the sequence $x=(t_i)_{i=0}^{\infty}$ is defined as

$$\omega(x, r)=\sup_{m\geq r}\sup_i|(\tau_m x)_i-t_i|,$$

where $r=0, 1, 2, \ldots$. Obviously, we have

$$\omega(x, r)=\sup_{m\geq r}\sup_{i\geq m}|t_{m+i}-t_i|$$