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Remarks on manifolds which admit locally free nilpotent Lie group actions

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0. Introduction

Let $\phi: G\times M\longrightarrow M$ be a smooth action of a connected Lie group G on a compact orientable manifold M. If for every point z of M the isotropy subgroup G_z is discrete, ϕ is said to be *locally free*. If the orbits of ϕ have codimension one, we call ϕ a *codimension one* action. Suppose that G is nilpotent and ϕ is a locally free codimension one action. Some dynamical properties of such an action ϕ and topological properties of M are stated in the paper [**HGM**]. We will consider this in detail. The object of this paper is to prove the following

THEOREM. *Let M be a connected closed orientable manifold. Suppose that M admits a locally free codimension one smooth action ϕ of a connected nilpotent Lie group G such that* i) *ϕ has no compact orbits and* ii) *the dimension of the commutator $[G, G]$ is one. Then M is homeomorphic to a nilmanifold i. e. the homogeneous space of a connected nilpotent Lie group.*

REMARK. (1) A compact nilmanifold always admits a locally free codimension one smooth action of a connected nilpotent Lie group which satisfies the above conditon i). (2) A Heisenberg group is a good example of a nilpotent Lie group which satisfies the above condition ii).

The theorem is a finer version of theorem (2.7) of [**HGM**] under the assumption ii).

Unless otherwise specified, we consider in the smooth (C^{∞}) category.

1. Unipotent flows on the space of lattices

Our method of proving the theorem is deeply concerned with characterization of a compact minimal set of a unipotent flow on the space of lattices. We describe it here.

Denote by $\mathscr{L}(k)$ the space of lattices in k-dimensional euclidean space $\boldsymbol{E}$ (cf. [C]). Fix a basis $\boldsymbol{v}_1, \cdots, \boldsymbol{v}_k$ of $\boldsymbol{E}$. Then every element $\boldsymbol{b}$ of a lattice Λ has a expression $\boldsymbol{b}=\sum_i(\sum_j b_{ij}m_j)\boldsymbol{v}_i$ where m_j's are integers and (b_{ij}) is