Hokkaido Mathematical Journal Vol. 17 (1988), p. 211-233

Completeness of the reduction of scalar, meromorphic differential equations of second order

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(Received April 9, 1987, Revised February 19, 1988)

1. Introduction

In this article we investigate meromorphic differential equations of the second order in scalar form. The study of these equations has a long history due to the countless applications of these equations in many branches of scientific research. Nevertheless the behavior of the solutions at singularities of the coefficients of such an equation is completely understood only in special cases (In the following the singularity under consideration is placed at infinity.). One of the central ideas of a systematic approach to this problem is to replace the general solution $x(z)$ of one differential equation by the combination $y(z)=t_1(z)x(z)+t_2(z)x'(z)$ where t_1, t_2 are simple functions, in the sense that they have at most a pole at infinity, such that $y(z)$ represents the general solution of another meromorphic differential equation. This introduces an equivalence relation among the differential equations, and the examination of the "essential" behavior of the solutions at infinity can now be reduced to the examination of one representative for each equivalence class. For that purpose, a practical choice of these representatives is required, and one hopes that they are as simple as possible.

This article is devoted to a solution of this problem and the investigation of related questions. The candidates are the special scalar equations which we defined and studied in [6]. We repeat the definition in section 2 where we also introduce closely related special matrix equations. The connection between the two kinds of equations is discussed in detail in section 4 (Proposition 1). It relies on some measure—theoretic relations between the coefficients and the zeros of a set of polynomials which are described in section 3 (Lemma 1). In section 5 we use these results to examine the equivalence of these special scalar equations (Theorem 2) and to show that they are as simple as possible by proving that they cannot be generated by a subcollection of equations with fewer parameters (Theorem 1). This is especially interesting since besides the singularity at infinity there appear further regular singularities in the finite plane which now turn out to be unavoidable. We will even identify a broad class of equations where the