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On differential invariants of integrable finite type linear differential equations

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§ 0. Introduction

The behavior of coefficients $a_1(t)$, $a_2(t)$, ..., $a_n(t)$ of a linear ordinary differential equation

$$\mathscr{R}: \left(\frac{d}{dt}\right)^n u + a_1(t)\left(\frac{d}{dt}\right)^{n-1} u + a_2(t)\left(\frac{d}{dt}\right)^{n-2} u + \ldots + a_n(t)u = 0$$

under a change of variables $\bar{t}=\phi(t)$, $\bar{u}=\lambda(t)u$ satisfying $\phi'(t)\neq 0$ and $\lambda(t)\neq 0$, was studied by Laguerre, Forsyth and others in the latter half of the 19 th century. Their fundamental results may be summarized as follows (cf. [5]):

i) There exists a change of variables which transforms $\mathscr{R}$ into a form $a_1(t)=a_2(t)=0$. (Such a form is called a Laguerre-Forsyth's canonical form of $\mathscr{R}$.)

ii) If a change of variables $(t, u)\longrightarrow(\bar{t}, \bar{u})$ transforms a canonical form into a canonical form, then there exists constants a, b, c, d and e such that

$$\phi(t)=\frac{at+b}{ct+d}, \qquad \lambda(t)=\frac{e}{(ct+d)^{n-1}}.$$

iii) For each canonical form of $\mathscr{R}$, let $\theta_p(t)(dt)^p$, $p=3, \ldots, n$ be the tensor fields defined respectively by

$$\theta_p(t)=\frac{(p-2)!\,p!}{(2p-3)!\,n!} \times\left\{\sum_{j=0}^{p-3}(-1)^j\frac{(2p-j-2)!\,(n-p+j)!}{(p-j-1)!\,j!}\left(\frac{d}{dt}\right)^j a_{p-j}(t)\right\}.$$

Then the definition of $\theta_p(t)(dt)^p$ does not depend on the choice of the Laguerre-Forsyth's canonical form of $\mathscr{R}$. Moreover $\theta_p(t)(dt)^p$ $p=3, \ldots, n$ form a fundamental system of invariants of $\mathscr{R}$.

The first purpose of this paper is to reformulate the classical Laguerre-Forsyth's theory of differential invariants of linear ordinary differential equations, by applying the E. Cartan's method. More precisely, we con-