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On positive solutions of quasi-linear elliptic equations

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ABSTRACT. In this note we prove the existence of positive solutions of the Dirichlet problem for a quasi-linear elliptic equation. Our boundary data belongs to L^2 and a corresponding solution is in a weighted Sobolev space.

1. Introduction.

Let $Q\subset R_n$ be a bounded domain with the boundary ∂Q of class C^2. In Q we consider the Dirichlet problem

(1) $Lu=-\sum_{i,j=1}^{n}D_i(a_{ij}(x, u)D_ju)+a_0(x)u=f(x, u)$ in Q,

(2) $u(x)=\phi(x)$ on ∂Q,

where ϕ is a non-negative function in $L^2(\partial Q)$.

Throughout this paper we make the following assumptions

(A) There is a positive constant γ such that

$$\gamma^{-1}|\xi|^2\leq\sum_{i,j=1}^{n}a_{ij}(x, u)\ \xi_i\xi_j\leq\gamma|\xi|^2$$

for all $\xi\in R_n$ and $(x, u)\in Q\times R$; $a_{ij}(x, u)=a_{ji}(x, u)$ $(i, j=1, \ldots, n)$ for all $(x, u)\in Q\times R$. Moreover, we assume that $a_{ij}(\cdot, \cdot)\in C(\bar{Q}\times R)$ $(i, j=1, \ldots, n)$ and for each $u\in R$, $a_{ij}(\cdot, u)\in C^1(\bar{Q})$ $(i, j=1, \ldots, n)$ and that there exist functions $A_{ij}\in C^1(\bar{Q})$ such that

$$\lim_{|u|\to\infty}a_{ij}(x, u)=A_{ij}(x) \text{ and } \lim_{|u|\to\infty}D_xa_{ij}(x, u)=D_xA_{ij}(x)\ (i, j=1, \ldots, n)$$

uniformly on $\bar{Q}$. Finally, the coefficient $a_0(x)$ is non-negative and belongs to $L^\infty(Q)$.

(B) The nonlinearity $f: Q\times R\to R$ satisfies the Carathéodory conditions, i. e.

(i) for each $u\in R$, the function $x\to f(x, u)$ is measurable in Q,

(ii) for each $x\in Q(a. e.)$, the function $u\to f(x, u)$ is continuous on R.

Further assumptions on f will be formulated later on.

In this note we use the notion of a generalized (weak) solution of (1) involving the Sobolev spaces $W_{loc}^{1,2}(Q)$, $W^{1,2}(Q)$ and $\mathring{W}^{1,2}(Q)$ (for the