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Construction of a parametrix for a weakly hyperbolic differential operator

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1. Introduction

In this paper we construct a microlocal parametrix for the Cauchy problem:

$$(1.1)\qquad \begin{cases} (D_t^2 - A(t, x, D_x))u(t, x) = 0, & (t, x) \in [0, T] \times \boldsymbol{R}^N \\ u(0, x) = g_0(x) & , \ x \in \boldsymbol{R}^N \\ \partial u/\partial t(0, x) = g_1(x) & , \ x \in \boldsymbol{R}^N \end{cases}$$

for small $T>0$, where $A(t, x, D_x)$ is a second order differential operator whose principal symbol $A_2(t, x, \xi)$ satisfies the following conditions:

$$(1.2)\qquad A_2(t, x, \xi) \geq 0 \text{ for } t \geq 0$$

$$(1.3)\qquad \frac{\partial A_2}{\partial t}(0, x, \xi) \neq 0 \text{ where } A_2(0, x, \xi) = 0, \ \xi \neq 0$$

Conditions (1. 2), (1. 3) imply that in a conic neighborhood of a point (x_0, ξ_0) for which $A_2(0, x_0, \xi_0) = 0$ and for small $t \geq 0$, we can factor A_2 as

$$(1.4)\qquad A_2(t, x, \xi) = (t + B(x, \xi))C(t, x, \xi)$$

with B homogeneous of degree 0, $B \geq 0$ and C is homogeneous of degree 2 with $C>0$. We recall that a parametrix for (1. 1), under conditions (1. 2), (1. 3) is already known in some particular cases.

When $B \equiv 0$, i. e. $A_2(t, x, \xi) = tC(t, x, \xi)$ (TRICOMI operator), a parametrix for (1. 1) was given by IMAI [2], who treated also the case when B does not depend on x (cfr. IMAI [3]). Furthermore, if B does not depend on ξ, a parametrix for (1, 1) has been constructed in SEGALA [6] (see also YOSHIKAWA [8]). We point out that ESKIN [1] and MELROSE [4] treated the "diffractive" case, where A_2 is factored as in (1. 4) and B satisfies $\nabla_\xi B \neq 0$ where $B=0$.

Here we will treat the general case when conditions (1. 2), (1. 3) are satisfied. Precisely, for every point $(x_0, \xi_0) \in T^*\boldsymbol{R}^N \backslash \{0\}$ where $A_2(0, x_0, \xi_0) = 0$, we will find a conic neighborhood Γ of (x_0, ξ_0) and for small $T>0$ construct a continuous operator