Hokkaido Mathematical Journal Vol. 17 (1988), p. 1~44

On the theory of discrete KMO-Langevin equations with reflection positivity (II)

Dedicated to Professor Seizo ITO on his sixtieth birthday

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(Received October 15, 1986, Revised September 28, 1987)

§ 1. Introduction

In a series of papers ([3]~[7]), the author has discussed real continuous-time stationary Gaussian processes $X(t)$ with reflection positivity and developed a theory of generalized Langevin equations describing the time evolution of such processes. His original aim was two-fold:

1) Deep understanding of a mathematical structure behind significant Kubo's fluctuation-dissipation theorem in statistical physics ([2]);
2) Applications of the theory as a useful model in various fields of science.

The first aim was achieved in [6] and [7]. Indeed, we established two different kinds of Langevin equations—the first (resp. second) KMO-Langevin equation with a white (resp. Kubo) noise as a random force, and proved the generalized fluctuation-dissipation theorems based on these equations. For the simplest process with Markovian property, i. e., for Ornstein-Uhlenbeck Brownian motion, these two kinds of equations take the same form, and the classical Einstein relation is valid. For a general $X(t)$, however, the situation turns out to be not so simple; the Einstein relation still holds if we use the equation of the second type, but does not hold in the case of the first type. So we raised in [7] a question how this interesting deviation from the Einstein relation can be measured experimentally in the remarkable case of Stokes-Boussinesque-Langevin equation of the first type.

With the second aim in mind, the author proceeded to investigate the discrete-time case in the previous paper [8], and the present paper is a continuation of [8]. Let us recall that we have considered in [8] stationary Gaussian time-series $X(n)$ with reflection positivity and established a discrete analogue of the results for the first KMO-Langevin equation obtained in [6] and [7]. As in the continuous-time case mentioned above, we have obtained the generalized Einstein relation based on the first KMO-Langevin

*) Partially supported by the Grant-in-Aid for Scientific Research of the Ministry of Education, Science and Culture of Japan.