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A characterization of some spreads of order q^3 that admit GL(2, q) as a collineation group

V. JHA and N. L. JOHNSON
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Introbuction

Let $\mathscr{F}\cong GF(q)$, $q=p^m$, be a field of matrices in $GL(3m, p)\cup\{O_{3m}\}$. Then $\pi=\boldsymbol{F}_p^{3m}\oplus\boldsymbol{F}_p^{3m}$ becomes a $\mathscr{G}=GL(2, \mathscr{F})$-module under the natural action of $\mathscr{G}\cong GL(2, q)$ on Π:

$$\underline{x}\oplus\underline{y}\longmapsto(\underline{x}a+\underline{y}b)\oplus(\underline{x}c+\underline{y}d)$$

whenever a, b, c, $d\in\mathscr{F}$ and $ad-bc\neq 0$.

We regard this action of $\mathscr{G}$ on π as defining the Desarguesian representation of $GL(2, q)$, of order q^3, because under this representation $GL(2, q)$ leaves invariant (several) Desarguesian spreads of order q^3, on π. Thus, if $\mathscr{M}\cong GF(q^3)$ is a field of matrices containing $\mathscr{F}$, then $\mathscr{M}$ defines a $\mathscr{G}$-invariant Desarguesian spread $\Gamma_{\mathscr{M}}$ whose components, apart from $Y=O_{3m}\oplus\boldsymbol{F}_p^{3m}$, have the generic form:

$$\{(\underline{x}, \underline{x}M):\ \underline{x}\in\boldsymbol{F}_p^{3m}\}\quad\text{for}\quad M\in\mathscr{M}.$$

Also there are often many non-Desarguesian spreads on π that are invariant under the Desarguesian representation of $GL(2, q)$. The first infinite families of such spreads were discovered by Kantor [7, 8], and later more examples were given in Bartolone-Ostrom [1]. Recently [5], we described a technique for constructing such spreads that yields all the abovementioned spreads of Kantor and Bartolone-Ostrom, and, in addition, yields many new examples. Our method allows one to construct a $\mathscr{G}$-invariant spread "$\pi_{\mathscr{O}}$", whenever one has a fixed-point-free collineation $\mathscr{O}\in P\Gamma L(3, q)$-$PGL(3, q)$ of the Desarguesian projective plane $PG(2, q)$. These "$\pi_{\mathscr{O}}$", which we shall call "generalized Desarguesian spreads", seem too numerous to classify as nonconjugate $\mathscr{O}$ usually yield nonisomorphic spreads.

The object of this note is to show that the only non-Desarguesian spreads invariant under the Desarguesian action of $GL(2, q)$ (of order q^3) are the generalized Desarguesian spreads.

THEOREM A. *Let π be a Desarguesian $GL(2, q)$-module of order q^3.*