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Zeros of integrals along trajectories of ergodic nonsingular flows

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§ 1. Introduction

Let $(X, \mathscr{B}, \mu)$ be a probability space. In 1976, Atkinson [1] proved that if T is an ergodic measure preserving automorphism of $(X, \mathscr{B}, \mu)$ then the following conditions are equivalent for f in $L_1(\mu)$:

(a) $\int f\, d\mu=0$.

(b) $\liminf_{n\to\infty}\left|\sum_{j=0}^{n} f(T^j x)\right|=0$ for almost all $x\in X$.

In 1987, Ullman [6] generalized Atkinson's theorem to noninvariant measures. That is, he considered an ergodic, conservative, nonsingular automorphism T of $(X, \mathscr{B}, \mu)$ and proved that the above condition (a) and the following (b′) are equivalent for f in $L_1(\mu)$.

(b′) $\liminf_{n\to\infty}\left|\sum_{j=0}^{n} f(T^j x)\cdot\frac{d\mu\circ T^j}{d\mu}(x)\right|=0$ for almost all $x\in X$.

In this note we will treat an ergodic, conservative, nonsingular flow of $(X, \mathscr{B}, \mu)$ and prove a corresponding continuous time result. The method of proof is different from that of Ullman. See also Schneiberg [5].

§ 2. Preliminaries and the theorem

From now on, let $\{T_t\}=\{T_t : -\infty<t<\infty\}$ be a measurable flow of nonsingular automorphisms of $(X, \mathscr{B}, \mu)$. All sets and functions introduced below are assumed to be measurable; and all relations are assumed to hold modulo sets of measure zero. Since each T_t is nonsingular, the Radon-Nikodym theorem can be applied to define a function $w_t=\frac{d\mu\circ T_t}{d\mu}$ in $L_1(\mu)$ such that

(1) $$\int_A w_t d\mu=\mu(T_t A) \text{ for all } A\in\mathscr{B},$$

and let us put