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A Bochner type theorem for compact groups*)

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Introduction

Let G be a compact abelian group and Γ_0 be a fixed subsemigroup of the dual group $\Gamma=\hat{G}$ of G. It is well known that in the case when G is the unit circle S^1 and $\Gamma_0=\boldsymbol{Z}_+$ any complex Borel measure $d\mu$ on G with zero nonpositive Fourier-Stieltjes coefficients $c_{-n}=\int_0^{2\pi} e^{int}d\mu(t)$, $n\in\boldsymbol{Z}_+$, is absolutely continuous with respect to the Haar (i. e. Lebesgue) measure $d\sigma$ on $G=S^1$. This is exactly the famous F. and M. Riesz theorem for analytic measures on the unit circle (e. g. [1]). In the sequel we shall use the following

DEFINITION 1. A pair (G, K) of a compact abelian group G and a subset K of its dual group $\Gamma=\hat{G}$ is said to be a Riesz pair if every finite Borel measure $d\mu$ orthogonal to K (i. e. $\int_G \chi(x)d\mu(x)=0$ for any $\chi\in K$) is absolutely continuous with respect to the Haar measure $d\sigma$ on G.

The F. and M. Riesz theorem says that $(S^1, \boldsymbol{Z}_+)$ is a Riesz pair. As shown by S. Koshi and H. Yamaguchi [3] in the case when $\Gamma_0\cup\Gamma_0^{-1}=\Gamma$ and $\Gamma_0\cap\Gamma_0^{-1}=\{1\}$ an analogue of F. and M. Riesz theorem for analytic measures on a compact connected group G does not hold unless $G=S^1$ and $\Gamma_0=\boldsymbol{Z}_+$ (or $\boldsymbol{Z}_-$). A theorem by I. Glicksberg [2] says that (S^1, Γ_0) is a Riesz pair for any subsemigroup Γ_0 of $\boldsymbol{Z}$, such that $\Gamma_0-\Gamma_0=\boldsymbol{Z}$. Consequently any finite complex Borel measure on S^1 that is orthogonal to such $\Gamma_0\subset\boldsymbol{Z}$ and is singular with respect to the Haar measure on S^1 coincides with the zero measure on S^1. On the other hand according to Bochner's theorem (e. g. [1]) (T^2, K) is a Riesz pair, where T^2 is the two dimensional torus and K is the complement in $\boldsymbol{Z}^2=\hat{\boldsymbol{T}}^2$ of a plane angle less then 2π edged at the origin. Here we extend Glicksberg's theorem and give a general construction of Riesz pairs that generalizes the Bochner's one.

1. Low-complete subsets of partially ordered sets

Let G be a compact abelian group. If Γ_0 is a subsemigroup of its dual

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