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Seminormal composition operators on L^2 spaces induced by matrices

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Abstract

The present paper deals with bounded seminormal composition operators on L^2 spaces induced by invertible linear transformations of the d-dimensional Euclidean space $\boldsymbol{R}^d$. A class of density functions on $\boldsymbol{R}^d$ for which seminormal composition operators can be completely characterized in terms of their symbols is distinguished. The spectrum and some related topics are discussed for particular case of Gaussian density function.

1. Introduction

Operators of the form $C_A f = f \circ A$ acting on certain function spaces are called composition operators (cf. [4], [12]). The present paper deals with a special class of composition operators induced by linear transformations of $\boldsymbol{R}^d$, acting on the function space $L^2(\boldsymbol{R}^d, r(x)dx)$, where r is a positive density function on $\boldsymbol{R}^d$. The most satisfactory description of seminormal composition operators of this form appears when the density function r is given by $r(\cdot)=\phi(\|\cdot\|^2)$, where $\|\cdot\|$ is a Hilbert norm on $\boldsymbol{R}^d$ and ϕ is a continuous function operating on positive definite matrices (i. e.) if $(a_{i,j})$ is a positive definite real $n\mathrm{x}n$ matrix, then the matrix $(\phi(a_{i,j}))$ is also positive definite). By the Schoenberg theorem [15] (see also [1] and [14]) such a function ϕ must have a power series representation with all non-negative Taylor's coefficients at 0. It turns out that subnormality of C_A does not depend on the function ϕ, though it depends on the transformation A. Namely C_A is subnormal if and only if A is normal in $(\boldsymbol{R}^d, \|\cdot\|)$. This fact enables us to describe the spectrum of C_A in particular case when $\phi=\exp$ and $\|\cdot\|$ is the usual norm on $\boldsymbol{C}^d$. Section 5 deals with another class of composition operators induced by restrictions of linear transformations of $\boldsymbol{R}^d$. In this case some new phenomena appears.

Recall that an operator C is said to be: *hyponormal* if $C^*C-CC^*\geq 0$, *subnormal* if it is a restriction of a normal operator, *quasinormal* if C commutes with C^*C. C is called *cohyponormal* (resp. *cosubnormal*, *co-*